Reading group on Infinity categories

Term 1, A.Y. 2020/2021

The goal of this reading group is to introduce to the formalism of ∞-categories (also called quasi-categories), as extensively developed by Joyal and Lurie. The Main source we will follow are Moritz Groth notes: "A short course on ∞-categories" (numbered theorems and sections refer to these notes). Complementary sources for proofs and more details are Charles Rezk, "stuff about quasi-categories" and the Kerodon website, in addition of course to Higher topos theory and Higher algebra of Lurie, all of them available on-line. We will divide the sections of Groth's notes in the following talks:

Section 1: Basics on ∞-categories(2 talks)

- Talk 1 (Week 2): Motivations: ∞-categories as generalisations of both ordinary categories and topological spaces. Brief recap and notation of simplicial sets. Nerve construction, definition of an ∞-category.
- Talk 2 (week 3): Homotopy category of an ∞-category, ∞-groupoids are Kan complexes (take as given technical proposition). Simplicial categories, coherent nerve. Corollary 1.39 and first examples of ∞-categories (examples 1.40). mention quickly model structures if time permits.

Section 2: Basic constructions with ∞-categories(2 talks)

- Talk 3 (week 4)(2.1, 2.2): proposition 2.5: The space of functors $\mathrm{Fun}(K, C)$ with C an ∞-category is an ∞-category. The ∞-category of ∞-categories. Join construction, left and right cones. The join of two ∞-categories is an ∞-category.
- Talk 4 (week 5): Slice construction. Initial and terminal objects, in particular corollary 2.24 and proposition 2.23. Limits and colimits in ∞-categories: pushouts and pullbacks as important examples.

Section 4: Monoidal ∞-categories(2 talks)

- Talk 5 (week 6)(4.1, 4.2): Subsection 4.1 as motivations, Segal maps and Segal condition. Cocartesian fibration, definition of a monoidal ∞-category, examples.

- Talk 6 (week 7): algebra objects, monoidal functors, definition of a symmetric monoidal ∞-category, ∞-category of commutative algebra objects.

Section 3: Presentable ∞-categories (1 talk)

- Talk 7 (week 8): Presentable ∞-categories, adjoint functor Theorem (Theorem 3.19).

Section 5: stable ∞-categories

- Talk 8 (week 9)(5.1, 5.2): Pointed ∞-categories, exact triangles, suspension functor , equivalent conditions for an ∞-category to be stable. Fiber and cofiber sequences. Stabilization, prespectrum and spectrum objects. The ∞-category of spectra is stable and presentable.

- Talk 9 (week 10): Tensor product of presentable ∞-categories. Smash product monoidal structure on spectra. The ∞-category of $\mathbb{E}_\infty$-ring spectra. The ∞-category of $\mathbb{A}_\infty$-ring spectra.