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↑ also me, but use the first one!

Advice and Feedback Hours: Fridays 11am – 1pm, Zeeman

https://warwick.ac.uk/fac/sci/maths/people/staff/cavaliere/ec119

$A = \{x \in \mathbb{N} : x^2 = 4\} = \{2\}$

$B = \{x \in \mathbb{Z} : x^2 = 4\} = \{-2, 2\}$

$C = \{x \in \mathbb{N} : -1 \leq x \leq 5\} = \{1, 2, 3, 4, 5\}$

$D = \{5x : x \in \mathbb{Z}, -3 < x < 2\} = \{-10, -5, 0, 5\}$

$E = \{x \in \mathbb{Q} : x^2 = 2\} = \emptyset$

$F = \{x \in \mathbb{R} : x^2 = 2\} = \{-\sqrt{2}, \sqrt{2}\}$

$G = \{\sqrt{x} : x \in \mathbb{N}, x < 5\} = \{1, \sqrt{2}, \sqrt{3}, 2\}$

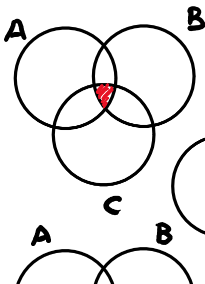
$A \cap B = \{x : x \in A \text{ and } x \in B\}$

$A \cup B = \{x : x \in A \text{ or } x \in B\}$

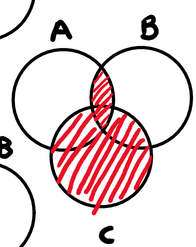
$A \setminus B = \{x : x \in A \text{ and } x \notin B\}$

$(A - B)$

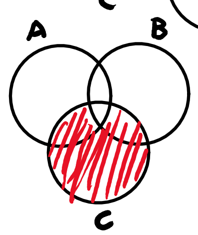
$(A \cap B) \cap C$



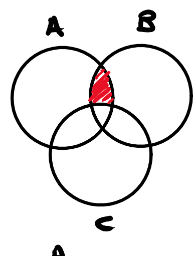
$(A \cap B) \cup C$



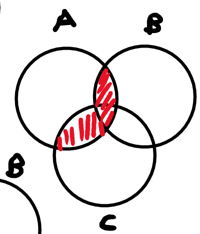
$C \setminus (A \cap B)$



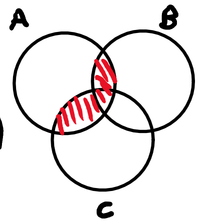
$(A \cap B) \setminus C$



$A \cap (B \cup C)$



$(A \cap B) \cup (A \cap C)$



Proposition 1.22 *The second distributive law*

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

holds, for any sets A, B and C.

Proof We begin by showing that $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$. Let $x \in A \cup (B \cap C)$. Then there are two possibilities: Either $x \in A$ or $x \notin A$.

(i) If $x \in A$, then obviously x must also lie in $(A \cup B) \cap (A \cup C)$.

(ii) If $x \notin A$ then it must be the case that $x \in B \cap C$, so that both

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$X \subseteq Y,\ Y \subseteq X \Rightarrow X = Y$

1. Show $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$

Proof We begin by showing that $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$. Let $x \in A \cup (B \cap C)$. Then there are two possibilities: Either $x \in A$ or $x \notin A$.

(i) If $x \in A$, then obviously x must also lie in $(A \cup B) \cap (A \cup C)$.
(ii) If $x \notin A$ then it must be the case that $x \in B \cap C$, so that both $x \in B$ and $x \in C$. Hence $x \in (A \cup B) \cap (A \cup C)$.

Therefore $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$.

We must now prove the converse, namely that $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$. Let $x \in (A \cup B) \cap (A \cup C)$. Again, there are two possibilities to consider:

(i) If $x \in A$, then clearly $x \in A \cup (B \cap C)$.
(ii) If $x \notin A$, then it must be the case that $x \in B \cap C$, and hence both $x \in B$ and $x \in C$.

So in either case, $x \in A \cup (B \cap C)$, hence $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$. Therefore $(A \cup B) \cap (A \cup C) = A \cup (B \cap C)$. $\square$[6]

$x = y, \ y = x \Rightarrow x = y$

1. Show $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$

Let $x \in A \cap (B \cup C)$.

$x \in A$ and $x \in B \cup C$

If $x \in B$, then $x \in A \cap B$

so $x \in (A \cap B) \cup (A \cap C)$.

If $x \notin B$, then $x \in C$,

so $x \in A \cap C$,

so $x \in (A \cap B) \cup (A \cap C)$.

Therefore, $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$

2. Show $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$