Set Theory

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Week 3

Class Content

Question 1

List the elements of the following sets:

1. $A = \{x \in \mathbb{N} : x^2 = 4\}$
2. $B = \{x \in \mathbb{Z} : x^2 = 4\}$
3. $C = \{x \in \mathbb{N} : -1 \leq x \leq 5\}$
4. $D = \{5x : x \in \mathbb{Z}, -3 < x < 2\}$
5. $E = \{x \in \mathbb{Q} : x^2 = 2\}$
6. $F = \{x \in \mathbb{R} : x^2 = 2\}$
7. $G = \{\sqrt{x} : x \in \mathbb{N}, x < 5\}$

Definition. *The* **intersection** *of a set A and a set B is the set $A \cap B = \{x : x \in A \text{ and } x \in B\}$.*

Definition. *The* **union** *of a set A and a set B is the set $A \cup B = \{x : x \in A \text{ or } x \in B\}$.*

Definition. *The* **complement** *of a set A relative to a set B is the set $A \setminus B = \{x : x \in A \text{ and } x \notin B\}$.*

Question 2

Let A, B and C be sets. Draw a Venn diagram showing the following sets:

1. $(A \cap B) \cap C$
2. $(A \cap B) \cup C$
3. $C \setminus (A \cap B)$
4. $(A \cap B) \setminus C$
5. $A \cap (B \cup C)$
6. $(A \cap B) \cup (A \cap C)$

Question 3

Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ for any sets A, B and C (the first distributive law).

Additional Questions

1. Write each of the following sets in set-builder notation (describing the set by giving a property that the elements must satisfy):

   (a) $\{-1, -2, -3, \dots\}$

   (b) $\{1, 3, 5, 7\}$

   (c) $\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots\}$

   (d) $(0, 1) \cap \mathbb{Q}$

   (e) $[-10, 10] \cap \mathbb{N}$

   Can you think of another way of writing them?

2. (a) Let A, B and C be sets. Which of the following is always true?

      i. $A \setminus (B \setminus C) = (A \setminus B) \cup C$

      ii. $A \setminus (B \cup C) = (A \setminus B) \setminus C$

      iii. $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$

   (b) For each statement which is true, give a proof. For each statement which is false, give an example of sets A, B and C such that the statement is false.