Please tick the register!

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also me, but use the first one!

Advice and Feedback Hours: Fridays 12pm – 1pm, Zeeman
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Vectors: $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 3 \\ -7 \\ 5 \end{pmatrix}$, $(2, 5)$

Matrices: $\begin{pmatrix} 1 & 5 \\ -8 & 0 \end{pmatrix}$, $\begin{pmatrix} 5 & 1 \\ 4 & -2 \\ 0 & -7 \end{pmatrix}$

Matrix Multiplication

$m \times p$ matrix A
$p \times n$ matrix B

The product AB is the $m \times n$ matrix where

$$(AB)_{ij} = \sum_{k=1}^{p} A_{ik} B_{kj}$$

ith row,
jth column

e.g. $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 0 & -3 \\ -9 & 6 \end{pmatrix}$

$$AB = \begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix} \overset{\begin{pmatrix}0 & -3\\ -9 & 6\end{pmatrix}}{\begin{pmatrix}-18 & 9\\ -36 & 15\end{pmatrix}} \qquad BA = \begin{pmatrix}-9 & -12\\ 9 & 6\end{pmatrix}$$

Transpose

$m \times n$ matrix A

The transpose is the $n \times m$ matrix A^T where $(A^T)_{ij} = A_{ji}$

e.g. $A = \begin{pmatrix}1 & 0 & -4\\ -5 & 2 & 8\end{pmatrix}$, $A^T = \begin{pmatrix}1 & -5\\ 0 & 2\\ -4 & 8\end{pmatrix}$

3 Let

$$A = \begin{bmatrix}7 & -10 & 2\\ 5 & 1 & 0\end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix}1 & 8 & -3\\ 5 & -3 & 2\\ 1 & -2 & 4\end{bmatrix}$$

2x3   3x3

Which of the following expressions are valid? Calculate those that are.

(a) A^T **(c)** BA **(e)** $B^T A^T$

(b) AB **(d)** BA^T **(f)** A^2

a) $\begin{pmatrix}7 & 5\\ -10 & 1\\ 2 & 0\end{pmatrix}$

c) Invalid

e) $\begin{pmatrix}-41 & 10\\ 82 & 37\\ -33 & -13\end{pmatrix}$

b) $\begin{pmatrix}-41 & 82 & -33\\ 10 & 37 & -13\end{pmatrix}$

d) $\begin{pmatrix}-79 & 13\\ 69 & 22\\ 35 & 3\end{pmatrix}$

f) Invalid

Linear Map

A function $f: \mathbb{R}^m \to \mathbb{R}^n$ is a linear map if for all $\underline{u}, \underline{v} \in \mathbb{R}^m$, $\lambda \in \mathbb{R}$

$f(\underline{u}+\underline{v}) = f(\underline{u}) + f(\underline{v})$
$f(\lambda \underline{v}) = \lambda f(\underline{v})$

e.g. $f, g, h: \mathbb{R}^2 \to \mathbb{R}$ where $f(x,y) = x+y$ ✓
$g(x,y) = x-1$ ✗
$h(x,y) = 0$ ✓

Let $(a,b), (c,d) \in \mathbb{R}^2$ and $\lambda \in \mathbb{R}$.
$f(a+c, b+d) = \dots = f(a,b) + f(c,d)$
$f(\lambda a, \lambda b) = \dots = \lambda f(a,b)$

6 Show that the following transformations are linear.

(a) $f\colon \mathbb{R}^2 \to \mathbb{R}^3$ with $f(x,y) = (3x+2y, -x, 2y-x)$

(b) $g\colon \mathbb{R}^3 \to \mathbb{R}^2$ with $g(x,y,z) = (x+y+z, x-y+z)$

(c) $h\colon \mathbb{R}^2 \to \mathbb{R}^2$ with $h(x,y) = (-y, x)$

c) Let $(a,b), (c,d) \in \mathbb{R}^2$ and $\lambda \in \mathbb{R}$.

$$h(a,b) + h(c,d) = (-b, a) + (-d, c) = (-b-d, a+c)$$
$$h(a+c, b+d) = (-(b+d), a+c) = (-b-d, a+c)$$
$$\therefore h(a+c, b+d) = h(a,b) + h(c,d)$$

$$\lambda h(a,b) = \lambda(-b, a) = (-\lambda b, \lambda a) = h(\lambda a, \lambda b)$$

__Theorem__

There is a one-to-one correspondence between

There is a one-to-one correspondence between linear maps and matrices.

Matrix $A \longrightarrow$ Linear map $f(\underline{x}) = A\underline{x}$

column vectors

Linear map $f \longrightarrow$ Matrix $\left(f(\underline{e}_1) \ \dots \ f(\underline{e}_n) \right)$

$\underline{e}_1, \dots, \underline{e}_n$ basis of domain of f

e.g. $f: \mathbb{R}^3 \to \mathbb{R}^2$ where $f(x,y,z) = (x+2y, y-3z)$

basis

$\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}$

$\underline{i}, \underline{j}, \underline{k}$

$f\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$

$f\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}2\\1\end{pmatrix}$

$f\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}0\\-3\end{pmatrix}$

So the matrix corresponding to f is $\begin{pmatrix}1 & 2 & 0\\0 & 1 & -3\end{pmatrix}$

6 Show that the following transformations are linear.

(a) $f: \mathbb{R}^2 \to \mathbb{R}^3$ with $f(x,y) = (3x+2y, -x, 2y-x)$

(b) $g: \mathbb{R}^3 \to \mathbb{R}^2$ with $g(x,y,z) = (x+y+z, x-y+z)$

(c) $h: \mathbb{R}^2 \to \mathbb{R}^2$ with $h(x,y) = (-y, x)$

7 (a) Write down matrices representing the linear maps in the previous question, relative to the standard bases for $\mathbb{R}^2$ and $\mathbb{R}^3$.

(b) Use your answers to calculate matrices representing the linear maps $g \circ f$, $f \circ g$ and h^{-1}.

$f(\underline{x}) = A\underline{x}$

$g(\underline{y}) = B\underline{y}$

$$(g \circ f)(\underline{x}) = g(f(\underline{x})) = g(A\underline{x}) = BA\underline{x}$$