

# Analysis I - Monday 31st October 2016

## Exam-Style Question

- (a) Define what it means for a sequence  $(a_n)$  to converge to a limit  $a$ .  
Define what it means for the sequence to be bounded.  
Prove that every convergent sequence is bounded. [6]
- (b) Which of the following sequences converge? Justify your answers.  
Find the limit for those sequences which converge to a real number.
- (i)  $a_n = \frac{\sin(n)+\cos(n)}{\log n}$ , for  $n \geq 2$ .
- (ii)  $b_n = \frac{\log(n)}{3+\sin(n)+\cos(n)}$ , for  $n \geq 2$
- (iii)  $c_n = \frac{\sin(n)+2\log(n)}{\cos(n)+\log(n)}$ , for  $n \geq 2$  [6]
- (c) Given a sequence  $(a_n)$ , recall that the  $n$ th term  $a_n$  is called a *floor term* if for every  $m > n$  we have that  $a_m \geq a_n$ .  
Using this notation, show that every sequence has either an increasing or a strictly decreasing subsequence. [8]