Open Problems

Zdeněk Dvořák: Let $\mathcal{G}$ be a class of graphs of bounded expansion, does there exist c_k such that for all G in $\mathcal{G}$ there is an ordering of $V(G)$ which demonstrates for all k that $\mathrm{col}_k(G) \leq c_k$?

This is known to be true when $\mathcal{G}$ is closed under topological minors.

Zdeněk Dvořák: Define $\chi_{(t)}(G)$ as the minimum number of colours in a colouring such that the union of any $p \leq t$ colour classes has tree depth at most p. Are there nice bounds on $\chi_{(t)}(G)$? In particular, is $\chi_{(t)}(G)$ polynomial in t for planar graphs?

It is known that $\chi_{(t)}(G) \leq wcol_{2^t}(G) + 1$. If G does not contain K_r as a minor then $wcol_k(G) = O(k^{r-1})$.

Solved by Patrice Ossona de Mendez and Sebastian Siebertz: there are polynomial bounds for any class of graphs avoiding some minor.

Mirna Džamonja: Let $A \subseteq \mathbb{R}$, $\mathcal{F} \subseteq [A]^{<\mathbb{N}}$ is $\frac{1}{2}$-dense if for all $B \in [A]^{<\mathbb{N}}$, there is a $C \subseteq B$ such that $C \in \mathcal{F}$ and $|C| \geq \frac{1}{2}|B|$.

Suppose $\mathcal{F}$ is $\frac{1}{2}$-dense and closed under subsets. Does there exist $D \in [A]^{\mathbb{N}}$ so that for all $B \in [D]^{<\mathbb{N}}$, $B \in \mathcal{F}$?

The statement is false for countable sets: consider the set $\mathcal{F}$ of subsets X of $\mathbb{N}$ such that $|X| \leq \min X$. It is also known to be false in ZFC with the continuum hypothesis.

Mirna Džamonja: Is $\mathrm{MSO}(\mathbb{R}, <)$ with set quantification restricted to Borel sets decidable?

Shelah proved in 1985 that MSO($\mathbb{R}, <$) is undecidable with arbitrary set quantification; on the other hand, MSO($\mathbb{Q}, <$) is decidable.

Agelos Georgakopoulos: For which notions of convergence do uniform random graphs on n vertices from a proper minor closed class of graphs, converge? Is it true in FO^{local}?

It is known that the uniformly random n-vertex planar map converges in the Benjamni-Schramm sense.

Jan Obdržálek: If D is a graph class interpretable in a class of nowhere dense graphs, then the FO model checking for D is in FPT.

This is known to be true for graph classes interpretable in a class of graphs with bounded maximum degree.

Patrice Ossona De Mendez: Let H be a connected graph, the H-colouring problem asks whether a graph is homomorphic to H. Consider Erdős-Rényi random graph $G \in G(n, \frac{d}{n})$; say the H-colouring problem is non-trivial if $\epsilon \leq \mathbb{P}(G \text{ has a } H \text{ colouring}) \leq 1 - \epsilon$ for some $\epsilon > 0$.

Is it true that the following are equivalent?

- The H-colouring problem has a coarse threshold, i.e., there exists a non-trivial interval of d's such that the H-colouring problem is non-trivial.
- For all $\varepsilon > 0$ and d such that the problem is non-trivial, there is a polynomial time algorithm for the H-colouring problem that works with probability at least $1 - \epsilon$.
- Either H is bipartite or for all d such that the problem is non-trivial there exists odd k such that $G \to H$ if and only if $C_k \not\to G$ with high probability.

Sylvain Schmitz A conjecture by Sławomir Lasota published in *Decidability Border for Petri Nets with Data: WQO Dichotomy Conjecture* (Proc. Petri nets 2017, vol. 9698 of Lect. Notes Comput. Sci., pp. 20–36): Let A be a countable relational structure over a finite vocabulary and $\mathrm{Age}(A, \leq)$ be its set of finite induced substructures, where $\leq$ is the induced substructure order.

Assuming A is homogeneous and has the amalgamation property and under some basic effectiveness assumptions, is the coverability problem in data Petri nets over A decidable if and only if $\mathrm{Age}(A, \leq)$ is a well-quasi-ordered?

Szymon Torunczyk: Let $\mathcal{G}$ be a class of graphs closed under induced subgraphs. If $\mathcal{G}$ is not NIP then FO model checking is not in FPT for $\mathcal{G}$. A class $\mathcal{G}$ is not NIP if there first order formula $\varphi(x_1, \ldots, x_k, y_1, \ldots, y_k)$ such that for every n there exists a graph G in $\mathcal{G}$ such that the bipartite graph $\varphi(G)$ with the vertex set $V(G)^k \cup V(G)^k$ contains a powerset graph of order n, i.e., it has a high VC dimension.

If $\mathcal{G}$ closed under subgraphs and not nowhere dense, then FO model checking is not in FPT for $\mathcal{G}$.

Jan Volec: Does there exist $c > 0$ such that for every simple directed graph D with average outdegree d we have $\alpha_A(D) \geq c\frac{n}{d} \log_2 d(1 - o(1))$? Here, $\alpha_A(D)$ stands for the number of vertices of the largest acyclic subgraph of D.

A result of Shearer from 1988 asserts this to be true for triangle-free graphs. It is also easy to prove for tournaments with a larger multiplicative constant.