Def: A graphon is an integrable function $W: [0,1]^2 \rightarrow \mathbb{R}_+$ such that $W(x,y) = W(y,x)$ for all $x, y \in [0,1]$

Embeding Graphs into Graphons: Given G on $[n]$, let $I_1, \ldots, , I_n$ be adjacent intervals of lengths $1/n$. Define $W^G: [0,1]^2 \rightarrow [0,1]$ by setting W^G to $A_{ij}(G)$ on $I_i \times I_j$ where $A(G)$ is the adjacency matrix of G

Cut norm and distance

$$\|W\|_\square = \sup_{S,T\subset[0.1]} \left| \int_{S\times T} W(x,y)dxdy \right|$$

$$d_\square(W,W') = \|W - W'\|_\square$$

$$\delta_\square(W,W') = \inf_\sigma d_\square(W^\sigma, W')$$

where the sup goes over measurable subsets $S, T \subset [0.1]$, the inf goes over invertible functions $\sigma: [0,1] \rightarrow [0,1]$ s.th. both σ and its inverse are measure preserving, and $W^\sigma(x,y)) = W(\sigma(x), \sigma(y))$.

Def.: G_n converges to W iff $\delta_\square(G_n, W) \rightarrow 0$

Thm [LS] The L^∞ ball of graphons equipped with the cut metric $\delta_\square$ is a compact metric space. In particular, every graph sequence has a convergent subsequence.