Problem Set 2 - from weak regularity to strong regularity

1. Prove the Frieze–Kannan weak regularity lemma. That is, show that for any $\epsilon > 0$, there is a partition $V(G) = X_1 \cup \dots X_k$ into $k \leq K$ parts such that for all $A, B \subseteq V(G)$,

$$|e(A,B) - \sum_{1 \leq i,j \leq k} d(X_i, X_j)|A \cup X_i||B \cap X_j|| \leq \epsilon |V|^2.$$

2. The Frieze–Kannan regularity lemma was developed to give an approximation algorithm for the max cut of a dense graph. How might this work?

3. Show that if a partition is weak ϵ-regular than any refinement is weak 2ϵ-regular. Show that this implies that we may assume the Frieze–Kannan regularity partition is equitable.

4. Is it true that if a partition is ϵ-regular, in the usual sense, then any refinement is 2ϵ-regular?

5. Use the strong regularity lemma to prove the induced removal lemma: for any $\epsilon > 0$, there exists $\delta > 0$ such that any graph on n vertices containing at most $\delta n^{v(H)}$ copies of H may be made induced-H-free by changing at most ϵn^2 edges.

6. Why is it not sufficient to remove edges in the induced removal lemma?