READING SEMINAR ON CYCLOTOMIC SPECTRA AND CARTIER MODULES

All the numbers of sections and results referred here are from our main reference [AN21].

(1) Oct 7: Short overview and organisation.

(2) Oct 14: Equivariant spectra. We give a short introduction to the various type of equivariant spectra that we will use throughout the seminar.

(3) Oct 21: Cyclotomic Spectra, TR and TC. This talk defines the category of p-cyclotomic spectra as in [AN21], and the notions of TR and TC from the same paper (see § 2.1: 2.5, 2.6, 3.18). A discussion of the relationship with the classical notion of cyclotomic spectra motivating the definition of TR would be useful, as in [NS18, §II.3 and Prop II.4.6].

(4) Oct 28: The cyclotomic t-structure. The goal of the talk is to cover the material of § 2.1, in particular theorem 2.1 (but without insisting on the multiplicative structures). It will require an introduction to t-structures on stable ∞-categories as in Appendix A and [Lur19, §1.2.1], in particular the definition of the heart and the homotopy groups, and [Lur19, 1.4.4.11].

(5) Nov 4: Topological Cartier Modules I: Examples. The goal of the talk is to introduce the category of topological Cartier modules and give some examples. Define (topological) Cartier modules as in 3.1 and give the examples 3.2-3.4, with a focus on TR. Then sketch a construction of the ring of p-typical Witt vectors as a Cartier module, as in [Hes15, §1] (focusing on the truncation set $\{1, p, p^n, \dots, \}$). Finally define the category of topological Cartier modules as in 3.6, and 3.12.

(6) Nov 11: Topological Cartier Modules II: The t-structure. This talk should cover §3.2 particularly theorem 3.15 (one could omit par (ii) which will be covered in the next talk). This will require a sketch of the description of mapping spaces of 3.7-3.10.

(7) Nov 18: Topological Cartier modules and cyclotomic spectra. The goal is Thm 3.21. Give the construction of the functor of Example 3.5 and prove 3.17. Then give definition 3.20 and prove 3.21 (which involves treating 3.15(ii) and 3.19). Stress that the cyclotomic homotopy groups can now be described in terms of TR. If time allows sketch 3.22.

(8) Nov 25: The heart. This should treat section 3.4, in particular Thm 2.26 and 3.28-3.30.

(9) Dec 2: THH of perfect rings. The goal is to sketch a construction of THH and a proof of theorem 6.14 (following the original references provided).

(10) Dec 9: TBD. Either a discussion of Cartier/Dieudonné complexes and the de Rham-Witt complex of §6.2 or an overview of genuine cyclotomic spectra of §5.

References

[AN21] Benjamin Antieau and Thomas Nikolaus, *Cartier modules and cyclotomic spectra*, J. Amer. Math. Soc. **34** (2021), no. 1, 1–78. MR 4188814

[Hes15] Lars Hesselholt, *The big de Rham-Witt complex*, Acta Math. **214** (2015), no. 1, 135–207. MR 3316757

[Lur19] Jacob Lurie, *Higher algebra*, www.math.harvard.edu/ lurie/papers/HA.pdf, 2019.

[NS18] Thomas Nikolaus and Peter Scholze, *On topological cyclic homology*, Acta Math. **221** (2018), no. 2, 203–409. MR 3904731