

READING SEMINAR ON THE ADAMS SPECTRAL SEQUENCE AND FRANKE'S ALGEBRAICITY CONJECTURE

All the numbers of sections and results referred here are from our main reference [PP21].

- (1) Jan 12: Short overview and organisation.
- (2) Jan 19: Homology theories and Adams spectral sequences. Cover the content of Sections 2.1 and 2.2, with lemma 6.53 as example of adapted homology theory.
- (3) Jan 26: The Freyd envelope and epimorphisms. Give an overview of section 2.3, culminating in the statement of the universal property of the Freyd envelope of Theorem 2.51.
Prove the implications $(1) \Rightarrow (2) \Rightarrow (3)$ of Theorem 2.56 (in our seminar we always start with a given adapted homology theory)
Define epimorphism classes as in 3.1 and prove Proposition 3.19 (no need to treat Miller's spectral sequence because of 3.20). Sketch a proof of the implications $(3) \Rightarrow (1)$ and $(3) \Rightarrow (2)$ of Theorem 3.24.
- (4) Feb 2: The prestable Freyd envelope and perfect presheaves. Give definition 4.1 and 4.2 and sketch Prop 4.9 and get Cor 4.10. Explain Example 4.17 which connects the prestable Freyd with classical Freyd envelope. Sketch the proof of Theorem 4.26.
Define prestable enhancements of homology functors 4.38 and discuss 4.40 and 4.44. Sketch the universal properties of Theorem 4.46 and Theorem 4.49.
- (5) Feb 9: Prestable enhancements and thread structures. Define prestable enhancements of homology functors 4.38 and discuss 4.40 and 4.44. Sketch the universal properties of Theorem 4.46 and Theorem 4.49.
Give an overview of Section 4.4, aiming at Propositions 4.63 and 4.65.
- (6) Feb 16: Bounded and perfect derived categories. The goal of the talk is to cover the constructions of the derived categories and their universal properties of section 5.1 and 5.2. Give Definition 5.14 and cover Proposition 5.16 and the universal property of Theorem 5.18. Then carry out the analogous constructions of S5.2 to be able to give definition 5.32.
- (7) Feb 23: Homology adjunction and the thread structure.
Recall Definition 5.32 and treat the universal property of Theorem 5.35. Treating these results involves giving an overview of some of the other constructions of 5.1 and 5.2 needed in the proofs.
Then cover Section 5.3, focusing on Theorem 5.45.
- (8) Mar 2: Obstruction theory. Cover the content of Section 6.3, focusing of the construction of the tower and its main Theorem 6.33. Use without proof the results about hypercompletion from Section 6.2.
- (9) Mar 9: Proof Part 1: Bousfield adjunction. This talk should cover Sections 7.1, 7.2, 7.3, with Proposition 7.29 begin the main result. Use Lemma 8.1 as an example of Definition 7.11.

- (10) Mar 16: Proof Part 2: Monadicity. Cover Sections 7.4 and 7.5, concluding the proof of Franke's conjecture 7.56, and get Theorem 8.2 and Corollary 8.3 as corollaries.

REFERENCES

- [PP21] Patchkoria and Pstrgowski, *The adams spectral sequence and franke's algebraicity conjecture*, arXiv:2110.03669, 2021.