READING SEMINAR ON TOPOLOGICAL CYCLIC HOMOLOGY

Topological Hochschild homology and topological cylcic homology were first introduced in [Bök86] and [BHM93] in order to study algebraic K-theory via trace invariants analogous to Chern classes. This theory led to several calculations with repercussions in topology of manifolds as well as arithmetic geometry, in particular in p-adic Hodge theory and crystalline cohomology.

The aim of the seminar is to understand Nikolaus and Scholze's formalism for topological Hochschild homology THH and topological cyclic homology TC of [NS18]. The first part of the seminar will introduce the definitions of THH and TC, and the second part will focus on calculations.

(1) Oct 9: Short overview and organization.

(2) Oct 16: Introduction to ∞-categories: This is an informal and concise introduction to ∞-categories. It should introduce the language and the main concepts to be used in the seminar, without digging too deep into details. The talk should in particular discuss stable ∞-categories, functor categories (in particular G-objects), and the categories fs spaces and spectra.

(3) Oct 23: Ring spectra: This talk introduces symmetric monoida ∞-categories, operads and their algebras (in particular E_1 and E_∞-algebras). In particular this provides a definition of ring spectra and E_∞-ring spectra. The main reference is [Lur19] and [NS18, Appendix S]

(4) Oct 30: Construction of THH: The goal is to construct THH of an E_1 ring as a spectrum with S^1-action. The talk should include a discussion of cyclic objects [NS18, Appendix T], and the construction of THH of [NS18, III.2].

(5) Nov 6: THH($\mathbb{F}_p$). This talk explains Bökstedt calculation of THH($\mathbb{F}_p$). One can follow the original proof as explained in [Rep18], or the argument of [KN19].

(6) Nov 13: Cyclotomic spectra and TC: This talk should introduce cyclotomic spectra as in [NS18, II.1], and as main example the cyclotomic structure on THH as of [NS18, III.2] .

(7) Nov 20: TC($\mathbb{F}_p$). This talk should cover the materials of [NS18, IV.4], providing a full picture of what happens for $\mathbb{F}_p$.

The last two talks, Nov 27 and Dec 4, will go in the direction of one of the following applications:

- Loop spaces. This talk should cover the calculation of THH and TC of group-rings as in [NS18, IV.3]. This can be applied to the K-theoretic Novikov conjecture [BHM93].

- THH and Witt vectors: Relation between THH and Witt vectors [HM97] and the deRham-Witt complex [HM04].

- Bhatt-Morrow-Scholze [BMS19], the relationship between TC and crystalline and syntomic cohomology.

References

[BHM93]   M. Bökstedt, W. C. Hsiang, and I. Madsen, *The cyclotomic trace and algebraic K-theory of spaces*, Invent. Math. **111** (1993), no. 3, 465–539. MR 1202133 (94g:55011)

[BMS19]   Bhargav Bhatt, Matthew Morrow, and Peter Scholze, *Topological Hochschild homology and integral p-adic Hodge theory*, Publ. Math. Inst. Hautes Études Sci. **129** (2019), 199–310. MR 3949030

[Bök86]   Marcel Bökstedt, *Topological Hochschild homology*, preprint, 1986.

[HM97]   Lars Hesselholt and Ib Madsen, *On the K-theory of finite algebras over Witt vectors of perfect fields*, Topology **36** (1997), no. 1, 29–101. MR 1410465 (97i:19002)

[HM04]   _____, *On the De Rham-Witt complex in mixed characteristic*, Ann. Sci. École Norm. Sup. (4) **37** (2004), no. 1, 1–43. MR 2050204

[KN19]   Achim Krause and Thomas Nikolaus, *Bökstedt periodicity and quotients of dvrs*, arXiv:1907.03477, 2019.

[Lur19]   Jacob Lurie, *Higher algebra*, www.math.harvard.edu/ lurie/papers/HA.pdf, 2019.

[NS18]   Thomas Nikolaus and Peter Scholze, *On topological cyclic homology*, Acta Math. **221** (2018), no. 2, 203–409. MR 3904731

[Rep18]   Oberwolfach Report, *Arbeitsgemeinschaft: Topological cyclic homology*, www.mfo.de/occasion/1814, 2018.