ERRATUM for "Calculus of Variations" (Filip Rindler, Springer 2018)
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- p.13, l.15: Replace $u(x) = Rx + u_0$ by $y(x) = Rx + y_0$ ($y_0 \in \mathbb{R}^3$, R as stated).
- p.14, Fig. 1.4: Replace $2r$ (on left) by $2R$.
- p.24, l.10: For weak metrizability also boundedness of X is needed (but this also follows later from the coercivity).
- p.32, l.-8 (Example 2.12): Replace $\mathbb{R}^m$ by $\mathbb{R}^3$.
- p.38, Theorem 2.17: Add the assumption (iv) there is at least one $u \in \mathrm{W}^{1,p}_g(\Omega;\mathbb{R}^m)$ with $\mathcal{H}[u] = 0$.
- p.38, l.15 (Theorem 2.17): Replace $\int_\Omega f(x, u(x), \nabla u(x))\,\mathrm{d}x$ by $\int_\Omega f(x, \nabla u(x))\,\mathrm{d}x$.
- p.39, l.5: Replace f by F.
- p.40, l.1: Add the condition $\dim X < \infty$.
- p.42, l.1 (Example 2.26): Also assume that $\varphi \geq 0$ and $\varphi(0) = \inf \varphi = 0$.
- p.54, l.-10 (Proposition 3.9): It should additionally be assumed here that $|\mathrm{D}_Z \mathrm{D}_A f(x, u, A)| \leq C(1+|u|+|A|)$ for $Z \in \{x, u, A\}$ in order for $\mathrm{div}[\mathrm{D}_A f(x, u(x), \nabla u(x))]$ to be well-defined (in fact, in (3.6) this existence is assumed).
- p.57, l.2 (Theorem 3.11): This result also holds, with the same proof, for any weak solution $u_* \in \mathrm{W}^{1,2}_{\mathrm{loc}}(\Omega;\mathbb{R}^m)$ of the corresponding Euler–Lagrange equation (this is used in the bootstrapping argument on p.61).
- p.58, l.-4: $k \in \{1, \ldots, d\}$ (braces missing).
- p.63, l.-4: Replace $f\colon \mathbb{R}^{d\times d} \to \mathbb{R}$ by $f\colon \mathbb{R}^d \to \mathbb{R}$.
- p.65, l.-3 (Theorem 3.21): One also needs to assume $|\mathrm{D}_v f(x, v, A)|, |\mathrm{D}_A f(x, v, A)| \leq C(1+|v|^{p-1}+|A|^{p-1})$ for some $C > 0$ and $p \in [1, \infty)$ (see Remark 3.2).
- p.69, l.15: f needs to be *twice* continuously differentiable and we also need to require that $H(\bullet, \tau) \in \mathrm{W}^{1,2}(\Omega;\mathbb{R}^m)$ for every $\tau \in \mathbb{R}$.
- p.70, l.7 (Theorem 3.23): The growth bound should read (no p): $|\mathrm{D}_v f(x, v, A)|, |\mathrm{D}_A f(x, v, A)| \leq C(1 + |v| + |A|)$.
- p.76, l.11 (Example 3.31): Replace $\leq$ by $\geq$.
- p.78, l.6 (Exercise 3.2): Delete point (iii).
- p.84, l.13 (Lemma 4.3): Replace $K \subset \mathbb{R}^{m\times d}$ by $K \subset \mathbb{R}^N$.
- p.84, l.15 (Lemma 4.3): Replace (ν_j) by $(\nu^{(j)})$.
- p.85, l.-7 (Theorem 4.4): The family $(\nu_x)_{x\in\Omega} \subset \mathcal{M}^1(\mathbb{R}^N)$ is weakly* measurable *with respect to* κ.
- p.88, l.3 (proof of Lemma 4.3): Replace "$=$" by "$\leq$" in front of $\frac{1}{h}\int_\Omega \int_{\{A\in\mathbb{R}^N \,:\, |A|^{p/2} \geq h\}} h^2 \,\mathrm{d}\nu^{(j)}_x(A)\,\mathrm{d}x$.
- p.88, l.10 (proof of Lemma 4.3): Replace $\lim_{j\to\infty}$ by $\limsup_{j\to\infty}$ (it is only later seen to be a limit).
- p.92, l.2 (Lemma 4.7): Replace $\mathrm{C}_0(\Omega) \times \mathrm{C}_0(\mathbb{R}^N)$ by $\mathrm{C}_0(\Omega \times \mathbb{R}^N)$.
- p.93, l.-6 (Example 4.10): Ω is $(0,1)^2$ everywhere.
- p.97, l.4 (proof of Lemma 4.13): Replace $|\tau_k V_{j(k)}|$ by $|\tau_k V_{j(k)}|^p$.

- p.97, l.10 & l.-10 (proof of Lemma 4.13): Replace v_k by $v_{j(k)}$.
- p.97, l.-12 (proof of Lemma 4.13): Replace V_k by $V_{j(k)}$.
- p.97, l.-11 & p.98, l.3 (proof of Lemma 4.13): Replace $h \in \mathrm{C}_0(\mathbb{R}^m)$ by $h \in \mathrm{C}_0(\mathbb{R}^{m\times d})$.
- p.103, l.9 (Problem 4.8): Also require $\nu_x(\partial E) = 0$.
- p.111, l.4 (proof of Proposition 5.3): The display should read (lim sup added): $\limsup_{k\to\infty} \|\nabla v_{j,k}\|_{\mathrm{L}^\infty} \le \limsup_{k\to\infty} \|\nabla u_k\|_{\mathrm{L}^\infty} + |F| < \infty$.
- p.115, l.10 (proof of Lemma 5.8): Replace $u \mapsto M(\nabla u)$ by $u \mapsto \int_\Omega M(\nabla u)\, \mathrm{d}x$.
- p.115, l.15 (proof of Lemma 5.8): Replace "$\cdot$" by "$\otimes$".
- p.116, l.4 (proof of Lemma 5.8): Replace $M^{\neg k}_{\neg l}(\nabla u) = \cdots$ by $(-1)^{k+l} M^{\neg k}_{\neg l}(\nabla u) = \cdots$.
- p.117, l.11 (Lemma 5.10): Replace L^∞ by $\mathrm{W}^{1,\infty}$.
- p.117, l.-11 (proof of Lemma 5.10): Replace $\int_\Omega M^{\neg k}_{\neg l}(\nabla u_j)\psi\, \mathrm{d}x$ by $(-1)^{k+l} \int_\Omega M^{\neg k}_{\neg l}(\nabla u_j)\psi\, \mathrm{d}x$; same for the following display.
- p.117, l.10 (proof of Lemma 5.10): The density argument needs to be applied only after the displays $\int_\Omega M^{\neg k}_{\neg l}(\nabla u)\psi\, \mathrm{d}x$ and $-\sum_{l=1}^3 \int_\Omega \big[u^1 (\operatorname{cof} \nabla u)^1_l\big] \partial_l \psi\, \mathrm{d}x = \int_\Omega \det \nabla u\, \psi\, \mathrm{d}x$, respectively.
- p.118, l.-6 & l.-2 (proof of Lemma 5.11): Replace Ω by $B(0,1)$.
- p.120, l.-2 (proof of Theorem 5.13): Replace $v \in \mathbb{R}^n$ by $v \in \mathbb{R}^d$.
- p.120, l.10 (Theorem 5.13 (ii)): Replace $\operatorname{dist}(\nabla u_j(x), \{A, B\}$ by $\operatorname{dist}(\nabla u_j(x), \{A, B\})$.
- p.123, l.11 (proof of Proposition 5.14): $(u_j) \subset \mathrm{W}^{1,p}(\Omega; \mathbb{R}^m)$ is a generating sequence for ν.
- p.123, l.-3 (proof of Proposition 5.14): Replace $\mathrm{d}x$ by $\mathrm{d}y$.
- p.128, l.2 (Lemma 5.19): The convergence in (5.15) only holds if the sequence $(f(x, u_j, V_j))_j$ additionally is assumed to be uniformly L^1-bounded and equiintegrable (like in Theorem 4.1 (iii)) . However (and this is what we use later in the proof of Theorem 5.20), for Carathéodory integrands $f\colon \Omega \times \mathbb{R}^M \times \mathbb{R}^N \to \mathbb{R}$ satisfying the p-growth bound (5.14), it holds that

$$\liminf_{j\to\infty} \int_\Omega f(x, u_j(x), V_j(x))\, \mathrm{d}x \ge \int_\Omega \big\langle f(x, u(x), \cdot), \nu_x \big\rangle\, \mathrm{d}x.$$

- p.128, l.-11 (Theorem 5.20): Replace $f\colon \Omega \times \mathbb{R}^m \times \mathbb{R}^{m\times d} \to \mathbb{R}$ by $f\colon \Omega \times \mathbb{R}^m \times \mathbb{R}^{m\times d} \to [0, \infty)$.
- p.129, l.1 (Remark 5.21): Replace $q \in [1, p/(d-p))$ by $q \in [1, dp/(d-p))$.
- p.140, l.-3 (proof of Lemma 6.6): Replace C^1 by C^∞ (or C^2); also several other occurrences throughout this proof.
- p.140, l.-2 (proof of Lemma 6.6): Delete $(-1)^{k+l}$.
- p.141, l.2 (proof of Lemma 6.6): Delete $(-1)^{k+l}$.
- p.147, l.-9 (proof of Theorem 6.9): Replace $x' \in u(\Omega)$ by $x' \in u_*(\Omega)$.
- p.156, l.6/7 (proof of Lemma 7.2): Replace D by $B(0,1)$.
- p.159, l.-10 (proof of Theorem 7.5): A more complete proof is as follows:
  The functional $\mathcal{F}_*$ is weakly lower semicontinuous as the supremum of weakly lower semicontinuous functionals. Indeed, if $u_j \rightharpoonup u$ in X, then for all weakly lower semicontinuous $\mathcal{H}\colon X \to \mathbb{R}$ with $\mathcal{H} \le \mathcal{F}$,

$$\mathcal{H}[u] \le \liminf_{j\to\infty} \mathcal{H}[u_j] \le \liminf_{j\to\infty} \mathcal{F}_*[u_j].$$

Taking the supremum over all such $\mathcal{H}$, we see that $\mathcal{F}_*[u] \le \liminf_{j\to\infty} \mathcal{F}_*[u_j]$.

Let $(u_j) \subset X$ be a mininimizing sequence for $\mathcal{F}$. By the weak coercivity, we may assume that $u_j \rightharpoonup u_*$ in X. Then,

$$\inf_X \mathcal{F} \le \mathcal{F}[u_*] \le \liminf_{j\to\infty} \mathcal{F}_*[u_j] \le \liminf_{j\to\infty} \mathcal{F}[u_j] = \inf_X \mathcal{F}.$$

Hence, $\mathcal{F}_*$ attains its minimum, which is equal to the infimum of $\mathcal{F}$ over X. □

- p.166, l.7 (Example 7.10): The sentence should read: "However, since rank $B = 1$, we have that $g(\mathbf{P}(A + tB)) = 0$ is affine in $t \in \mathbb{R}$.".

- p.172 ff. (Theorem 7.15): The functional-analytic setup in the proof needs to be changed as follows: Let $\mathcal{M}_p(\mathbb{R}^{m\times d})$ for $p \in (1,\infty)$ denote the class of finite signed measures on $\mathbb{R}^{m\times d}$ with bounded p'th-order absolute moment, that is, $\mathcal{M}_p(\mathbb{R}^{m\times d}) := \{ \mu \in \mathcal{M}(\mathbb{R}^{m\times d};\mathbb{R}) \;:\; \int 1 + |A| \,\mathrm{d}\mu(A) < \infty \}$. Then, an integrand $h \in \mathbf{I}^p(\mathbb{R}^{m\times d})$ can be viewed as a linear functional on $\mathcal{M}_p(\mathbb{R}^{m\times d})$ via the duality pairing $\langle \mu, h\rangle := \int h \,\mathrm{d}\mu$ (where $\mu \in \mathcal{M}_p(\mathbb{R}^{m\times d})$). The functionals $h \mapsto \langle \,\bullet\,, h\rangle$ (where $h \in \mathbf{I}^p(\mathbb{R}^{m\times d})$) separate the points of $\mathcal{M}_p(\mathbb{R}^{m\times d})$, that is, for $\mu_1, \mu_2 \in \mathcal{M}_p(\mathbb{R}^{m\times d})$ with $\mu_1 \neq \mu_2$ there is an integrand $h \in \mathbf{I}^p(\mathbb{R}^{m\times d})$ such that $\langle \mu_1, h\rangle \neq \langle \mu_2, h\rangle$.

  Let τ_p be the weakest topology on $\mathcal{M}_p(\mathbb{R}^{m\times d})$ that makes all the functionals $h \mapsto \langle \,\bullet\,, h\rangle$ continuous. It is a general result of topology (see, e.g., Theorem 3.10 of [W. Rudin: Functional Analysis, McGraw–Hill, 1991]) that the topological space $(\mathcal{M}_p(\mathbb{R}^{m\times d}), \tau_p)$ is a locally convex topological vector space and its dual space $(\mathcal{M}_p(\mathbb{R}^{m\times d}), \tau_p)^*$ is given as $\mathbf{I}^p(\mathbb{R}^{m\times d})$ (via the above duality pairing).

  The set $\mathbf{GY}^p_{\mathrm{hom}}(F)$ then needs to be viewed as a subset of the space $\mathcal{M}_p(\mathbb{R}^{m\times d})$ (and not of $\mathbf{I}^p(\mathbb{R}^{m\times d})^*$ as before), where it is convex and τ_p-closed (it is not weakly*-closed in $\mathbf{I}^p(\mathbb{R}^{m\times d})^*$ because there is no tightness of the masses, e.g., for $\nu_j = (1-j^{-p})\delta_0 + (j^{-p}\delta_{-jA} + j^{-p}\delta_{jA})/2$ as $j \to \infty$, where A is a rank-one matrix). In Lemma 7.17 and the proof of Theorem 7.15 one thus needs to replace every occurrence of $\mathbf{I}^p(\mathbb{R}^{m\times d})^*$ by $\mathcal{M}_p(\mathbb{R}^{m\times d})$ (effectively moving from a dual to a pre-dual) and use the τ_p-topology instead of the weak* topology everywhere; the arguments are otherwise the same. As a result, Lemma 7.17 then needs to read as follows: "For any $F \in \mathbb{R}^{m\times d}$ the set $\mathbf{GY}^p_{\mathrm{hom}}(F)$ is convex and τ_p-closed in $\mathcal{M}_p(\mathbb{R}^{m\times d})$." The application of the Hahn–Banach separation theorem (for locally convex topological vector spaces; see Theorem 3.4 in *loc. cit.*) is then also with respect to the space $(\mathcal{M}_p(\mathbb{R}^{m\times d}), \tau_p)$ and its dual $\mathbf{I}^p(\mathbb{R}^{m\times d})$. [This corrected setup is due to Stefan Müller]

- p.174, l.13 (proof of Lemma 7.17): Replace $u_j \in \mathrm{W}^{1,p}(\Omega;\mathbb{R}^m)$ by $u_j \in \mathrm{W}^{1,p}(B(0,1);\mathbb{R}^m)$.

- p.178, l.16 (Theorem 7.18): Delete the second "convex".

- p.175, l.1 (proof of Theorem 7.15): The necessity proof also needs to explicitly invoke Proposition 5.14 to localize.

- p.222, l.5 (Lemma 8.32): Replace $\rightharpoonup$ by $\overset{*}{\rightharpoonup}$ (in $\mathcal{M}_{\mathrm{loc}}$).

- p.260, l.-3: Replace $K \subset \mathbb{R}^{3\times 12}$ by $K \subset \mathbb{R}^{3\times 2}$.

- p.278, l.-12 (Lemma 10.6): Replace μ_0 by μ.

- p.298, l.4 (Problem 10.4): Replace $y \in Q_n(x_0, r)$ by $y \in Q_n(0,1)$.

- p.326, l.16 (proof of Theorem 11.21): Replace $\ge \mu \|\nabla u_j\|_{\mathrm{L}^1}$ by $\ge \mu \cdot \limsup_{j\to\infty} \|\nabla u_j\|_{\mathrm{L}^1}$.

- p.372, l.-4: Replace $\liminf_{k\to\infty} \mathcal{F}_k[u_k] = \liminf_{k\to\infty} \inf_X \mathcal{F}_k$ by $\lim_{k\to\infty} \mathcal{F}_k[u_k] = \liminf_{k\to\infty} \inf_X \mathcal{F}_k$.

- p.394, l.-4 (eq. (13.29)): Add $\limsup_{\varepsilon\downarrow 0}$ after the first "$\le$".

- p.423, l.-7: Replace $\eta_\delta(x) := \frac{1}{\delta^d}\eta\big(\frac{x}{\delta^d}\big)$ by $\eta_\delta(x) := \frac{1}{\delta^d}\eta\big(\frac{x}{\delta}\big)$.

- p.426, l.-2 (Theorem A.36): f (not Mf) is Lipschitz on the set $\{M(|f| + |\nabla f|) < K\}$.