ERRATUM for "Calculus of Variations" (Filip Rindler, Springer 2018)
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- p.14, Fig. 1.4: $2r$ (on left) replaced by $2R$
- p.24, l.10: for weak metrizability also need boundedness of X (but this also follows later from the coercivity)
- p.32, l.-8 (Example 2.12): $\mathbb{R}^m$ replaced by $\mathbb{R}^3$
- p.54, l.-10 (Proposition 3.9): It should additionally be assumed here that $|\mathrm{D}_Z \mathrm{D}_A f(x,u,A)| \le C(1+|u|+|A|)$ for $Z \in \{x,u,A\}$ in order for $\operatorname{div}[\mathrm{D}_A f(x,u(x),\nabla u(x))]$ to be well-defined (in fact, in (3.6) this existence is assumed).
- p.57, l.2 (Theorem 3.11): This result also holds, with the same proof, for any weak solution $u_* \in \mathrm{W}^{1,2}_{\mathrm{loc}}(\Omega;\mathbb{R}^m)$ of the corresponding Euler–Lagrange equation (this is used in the bootstrapping argument on p.61).
- p.58, l.-4: $k \in \{1,\dots,d\}$ (parentheses missing).
- p.63, l.-4: $f\colon \mathbb{R}^{d\times d} \to \mathbb{R}$ replaced by $f\colon \mathbb{R}^d \to \mathbb{R}$.
- p.65, l.-3 (Theorem 3.21): One also needs to assume $|\mathrm{D}_v f(x,v,A)|, |\mathrm{D}_A f(x,v,A)| \le C(1+|v|^{p-1}+|A|^{p-1})$ for some $C>0$ and $p \in [1,\infty)$ (see Remark 3.2).
- p.69, l.15: f needs to be *twice* continuously differentiable and we also need to require that $H(\bullet,\tau) \in \mathrm{W}^{1,2}(\Omega;\mathbb{R}^m)$ for every $\tau \in \mathbb{R}$
- p.70, l.7 (Theorem 3.23): The growth bound should read (no p):
  $|\mathrm{D}_v f(x,v,A)|, |\mathrm{D}_A f(x,v,A)| \le C(1+|v|+|A|)$
- p.78, l.6 (Exercise 3.2): Delete point (iii).
- p.84, l.13 (Lemma 4.3): $K \subset \mathbb{R}^{m\times d}$ replaced by $K \subset \mathbb{R}^N$.
- p.84, l.15 (Lemma 4.3): (ν_j) replaced by $(\nu^{(j)})$.
- p.85, l.-7 (Theorem 4.4): The family $(\nu_x)_{x\in\Omega} \subset \mathcal{M}^1(\mathbb{R}^N)$ is weakly* measurable *with respect to* κ.
- p.92, l.2 (Lemma 4.7): $\mathrm{C}_0(\Omega) \times \mathrm{C}_0(\mathbb{R}^N)$ replaced by $\mathrm{C}_0(\Omega \times \mathbb{R}^N)$
- p.93, l.-6 (Example 4.10): Ω is $(0,1)^2$ everywhere
- p.97, l.4 (proof of Lemma 4.13): $|\tau_k V_{j(k)}|$ replaced by $|\tau_k V_{j(k)}|^p$
- p.97, l.10 & l.-10 (proof of Lemma 4.13): v_k replaced by $v_{j(k)}$.
- p.97, l.-12 (proof of Lemma 4.13): V_k replaced by $V_{j(k)}$.
- p.97, l.-11 & p.98, l.3 (proof of Lemma 4.13): $h \in \mathrm{C}_0(\mathbb{R}^m)$ replaced by $h \in \mathrm{C}_0(\mathbb{R}^{m\times d})$.
- p.103, l.9 (Problem 4.8): Also require $\nu_x(\partial E) = 0$.
- p.111, l.4 (proof of Proposition 5.3): The display should read (lim sup added):
  $\limsup_{k\to\infty} \|\nabla v_{j,k}\|_{\mathrm{L}^\infty} \le \limsup_{k\to\infty} \|\nabla u_k\|_{\mathrm{L}^\infty} + |F| < \infty$
- p.115, l.10 (proof of Lemma 5.8): $u \mapsto M(\nabla u)$ replaced by $u \mapsto \int_\Omega M(\nabla u)\,\mathrm{d}x$
- p.115, l.15 (proof of Lemma 5.8): "$\cdot$" replaced by "$\otimes$".
- p.116, l.4 (proof of Lemma 5.8): $M^{\neg k}_{\neg l}(\nabla u) = \cdots$ replaced by $(-1)^{k+l} M^{\neg k}_{\neg l}(\nabla u) = \cdots$
- p.117, l.11 (Lemma 5.10): L^∞ replaced by $\mathrm{W}^{1,\infty}$

- p.117, l.-11 (proof of Lemma 5.10): $\int_\Omega M^{\neg k}_{\neg l}(\nabla u_j)\psi \,\mathrm{d}x$ replaced by $(-1)^{k+l}\int_\Omega M^{\neg k}_{\neg l}(\nabla u_j)\psi \,\mathrm{d}x$; same for the following display
- p.117, l.10 (proof of Lemma 5.10): The density argument needs to be applied only after the displays $\int_\Omega M^{\neg k}_{\neg l}(\nabla u)\psi \,\mathrm{d}x$ and $-\sum_{l=1}^{3}\int_\Omega \big[u^1(\operatorname{cof}\nabla u)^1_l\big]\partial_l\psi \,\mathrm{d}x = \int_\Omega \det\nabla u\,\psi \,\mathrm{d}x$, respectively.
- p.118, l.-6 & l.-2 (proof of Lemma 5.11): Ω replaced by $B(0,1)$.
- p.120, l.-2 (proof of Theorem 5.13): $v \in \mathbb{R}^n$ replaced by $v \in \mathbb{R}^d$.
- p.120, l.10 (Theorem 5.13 (ii)): $\operatorname{dist}(\nabla u_j(x), \{A,B\}$ replaced by $\operatorname{dist}(\nabla u_j(x), \{A,B\})$.
- p.123, l.11 (proof of Proposition 5.14): $(u_j) \subset \mathrm{W}^{1,p}(\Omega;\mathbb{R}^m)$ is a generating sequence for ν.
- p.123, l.-3 (proof of Proposition 5.14): $\mathrm{d}x$ replaced by $\mathrm{d}y$
- p.128, l.2 (Lemma 5.19): The convergence in (5.15) only holds if the sequence $(f(x,u_j,V_j))_j$ additionally is assumed to uniformly L^1-bounded and equiintegrable (like in Theorem 4.1 (iii)) . However (and this is what we use later in the proof of Theorem 5.20), for Carathéodory integrands $f\colon \Omega\times\mathbb{R}^M\times\mathbb{R}^N\to\mathbb{R}$ satisfying the p-growth bound (5.14), it holds that

$$\liminf_{j\to\infty}\int_\Omega f(x,u_j(x),V_j(x))\,\mathrm{d}x \geq \int_\Omega \big\langle f(x,u(x),\,\cdot\,),\nu_x\big\rangle\,\mathrm{d}x.$$

- p.128, l.-11 (Theorem 5.20): $f\colon \Omega\times\mathbb{R}^m\times\mathbb{R}^{m\times d}\to\mathbb{R}$ replaced by $f\colon \Omega\times\mathbb{R}^m\times\mathbb{R}^{m\times d}\to[0,\infty)$
- p.129, l.1 (Remark 5.21): $q\in[1,p/(d-p))$ replaced by $q\in[1,dp/(d-p))$
- p.140, l.-3 (proof of Lemma 6.6): C^1 replaced by C^∞ (or C^2); also several other occurrences throughout this proof
- p.140, l.-2 (proof of Lemma 6.6): delete $(-1)^{k+l}$
- p.141, l.2 (proof of Lemma 6.6): delete $(-1)^{k+l}$
- p.147, l.-9 (proof of Theorem 6.9): $x'\in u(\Omega)$ replaced by $x'\in u_*(\Omega)$
- p.156, l.6/7 (proof of Lemma 7.2): D replaced by $B(0,1)$.
- p.159, l.-10 (proof of Theorem 7.5): A more complete proof is as follows:
  The functional $\mathcal{F}_*$ is weakly lower semicontinuous as the supremum of weakly lower semicontinuous functionals. Indeed, if $u_j\rightharpoonup u$ in X, then for all weakly lower semicontinuous $\mathcal{H}\colon X\to\mathbb{R}$ with $\mathcal{H}\leq\mathcal{F}$,

$$\mathcal{H}[u]\leq\liminf_{j\to\infty}\mathcal{H}[u_j]\leq\liminf_{j\to\infty}\mathcal{F}_*[u_j].$$

  Taking the supremum over all such $\mathcal{H}$, we see that $\mathcal{F}_*[u]\leq\liminf_{j\to\infty}\mathcal{F}_*[u_j]$.
  Let $(u_j)\subset X$ be a mininimizing sequence for $\mathcal{F}$. By the weak coercivity, we may assume that $u_j\rightharpoonup u_*$ in X. Then,

$$\inf_X\mathcal{F}\leq\mathcal{F}[u_*]\leq\liminf_{j\to\infty}\mathcal{F}_*[u_j]\leq\liminf_{j\to\infty}\mathcal{F}[u_j]=\inf_X\mathcal{F}.$$

  Hence, $\mathcal{F}_*$ attains its minimum, which is equal to the infimum of $\mathcal{F}$ over X. □
- p.166, l.7 (Example 7.10): The sentence should read: "However, since $\operatorname{rank}B=1$, we have that $g(\mathbf{P}(A+tB))=0$ is affine in $t\in\mathbb{R}$."
- p.174, l.13 (proof of Lemma 7.17): $u_j\in\mathrm{W}^{1,p}(\Omega;\mathbb{R}^m)$ replaced by $u_j\in\mathrm{W}^{1,p}(B(0,1);\mathbb{R}^m)$
- p.175, l.1 (proof of Theorem 7.15): The necessity proof also needs to explicitly invoke Proposition 5.14 to localize
- p.260, l.-3: $K\subset\mathbb{R}^{3\times 12}$ replaced by $K\subset\mathbb{R}^{3\times 2}$
- p.278, l.-12 (Lemma 10.6): μ_0 replaced by μ

- p.326, l.16 (proof of Theorem 11.21): $\geq \mu \|\nabla u_j\|_{\mathrm{L}^1}$ replaced by $\geq \mu \cdot \limsup_{j\to\infty} \|\nabla u_j\|_{\mathrm{L}^1}$
- p.423, l.-7: $\eta_\delta(x) := \frac{1}{\delta^d}\eta\big(\frac{x}{\delta^d}\big)$ replaced by $\eta_\delta(x) := \frac{1}{\delta^d}\eta\big(\frac{x}{\delta}\big)$.
- p.426, l.-2 (Theorem A.36): f (not Mf) is Lipschitz on the set $\{M(|f| + |\nabla f|) < K\}$