Reading Group on
Higher Zariski Geometry

Syllabus

Autumn 2025

This syllabus should be treated in a flexible way. If we want to spend more time on background material or a particular topic then we should do it. It is also an option to continue in term 2.

Week 1: Review of the definitions in classical tt-geometry, including Thomason's reconstruction theorem for a quasicompact quasiseparated scheme and the construction of the presheaf of tt-categories that assigns to an open subset of the Balmer spectrum the Verdier quotient by the objects supported away from the open subset. Explain why this is generally not a sheaf.
References: [Bal05], [Tho97], [Bal02, Section 5], [BF07]

Week 2: Overview over the higher Zariski geometry paper, motivation, main theorems. Recall some of the ∞-categorical background: Section 2A and ∞-topoi.
References: [ABCSS25, Section 1, 2A], [Lur09, Section 6]

Week 3: The ∞-category of commutative "2-rings" (that is, idempotent complete stably symmetric monoidal ∞-categories). Thick tensor ideals, localisations, rigidity and the interplay between the latter two.
References: [ABCSS25, Section 2B-2D]

Week 4: Geometries and locality, structures for a geometry.
References: [ABCSS25, Section 3A, 3B], [Lur11a, Section 1]

Week 5: Affine spectra with respect to a geometry, classical Zariski geometry in the context of Lurie's framework.
References: [ABCSS25, Section 3C, 3D], [Lur11a, Section 2], [Lur11b, Section 2]

Week 6: The Zariski geometry on 2-rings, comparison with the classical Balmer spectrum involving a digression on lattices and frames.
References: [ABCSS25, Section 4A, 4B], [Aok23, Section 3], [KP17]

Week 7: Locally 2-ringed ∞-topoi, comparison transformations, Thomason's theorem in higher Zariski geometry ([Che], not on the arxiv yet).
References: [ABCSS25, Section 4C-4E]

Week 8: Zariski descent and Mayer–Vietoris, the structure sheaf and full faithfulness of the absolute spectrum.
References: [ABCSS25, Section 5A-5B], [Aok25, Section 3]

Week 9: Zariski descent for modules, i.e. sheaves of modules on the absolute spectrum of a rigid 2-ring.
References: [ABCSS25, Section 5C]

Week 10: Support data and structures for the Zariski geometry on 2-rings: comparison to Balmer's original universal property of the spectrum of a tt-category as its final support datum.
References: [Bal05], [ABCSS25, Appendix A]

Week 11 (optional): Stalk-locality of the telescope conjecture, ideally with some background on the telescope conjecture and why it is interesting.
References: [ABCSS25, Appendix B]

References

[ABCSS25]   Ko Aoki, Tobias Barthel, Anish Chedalavada, Tomer Schlank and Greg Stevenson. *Higher Zariski Geometry.* 2025. arXiv: `2508.11621v1`.

[Aok23]   Ko Aoki. 'Tensor triangular geometry of filtered objects and sheaves'. In: *Math. Z.* 303.3 (2023), Paper No. 62, 27.

[Aok25]   Ko Aoki. *The sheaves–spectrum adjunction.* 2025. arXiv: `2302.04069v2`.

[Bal02]   Paul Balmer. 'Presheaves of triangulated categories and reconstruction of schemes'. In: *Math. Ann.* 324.3 (2002), pp. 557–580.

[Bal05]   Paul Balmer. 'The spectrum of prime ideals in tensor triangulated categories'. In: *J. Reine Angew. Math.* 588 (2005), pp. 149–168.

[BF07]   Paul Balmer and Giordano Favi. 'Gluing techniques in triangular geometry'. In: *Q. J. Math.* 58.4 (2007), pp. 415–441.

[KP17]   Joachim Kock and Wolfgang Pitsch. 'Hochster duality in derived categories and point-free reconstruction of schemes'. In: *Trans. Amer. Math. Soc.* 369.1 (2017), pp. 223–261.

[Lur09]   Jacob Lurie. *Higher topos theory.* Vol. 170. Annals of Mathematics Studies. Princeton University Press, Princeton, NJ, 2009, pp. xviii+925.

[Lur11a]   Jacob Lurie. *Derived Algebraic Geometry V: Structured spaces.* Available at: `https://www.math.ias.edu/~lurie/`. 2011.

[Lur11b]   Jacob Lurie. *Derived Algebraic Geometry VII: Spectral Schemes.* Available at: `https://www.math.ias.edu/~lurie/`. 2011.

[Tho97]   Robert W. Thomason. 'The classification of triangulated subcategories'. In: *Compositio Math.* 105.1 (1997), pp. 1–27.