1. Read, understand, and reproduce in your own words the proof of Theorem 16 (Theorem 2.12 in Robinson).

2. Let $A = \{x_j\} \cup \{0\}$ such that $x_j \to 0$ and $\delta_j = x_j - x_{j+1}$ is decreasing. Find an explicit expression for $\dim_{\mathrm{B}}(A)$.

3. Suppose that X is a compact subset of $\mathbb{R}^N$. Show that the collection of linear maps from $\mathbb{R}^N$ into $\mathbb{R}^k$ that satisfy

$$|Lz| > |z|^{1/\alpha}$$

for all $|z| < R$ for some $R > 0$ is open. [Consider the maps that satisfy $|Lz| > |z|^{1/\alpha}$ for all $2^{-k} \leq |z| \leq 2^{-(k+1)}$.]

4. One can adapt the proof of Theorem 35 to show that the set of linear maps $L\colon \mathbb{R}^N \to \mathbb{R}^k$ that are embeddings satisfying the Hölder property are dense in $B(\mathbb{R}^n, \mathbb{R}^k)$. See Theorem 4.3 in Robinson. Combine this with the result of Exercise 3 to show that a residual set of linear maps in $B(\mathbb{R}^n, \mathbb{R}^k)$ are embeddings with Hölder continuous inverses when $k > 2\dim_{\mathrm{B}}(X)$.

5. Investigate the optimality (or otherwise) of the result of Lemma 42.

6. Show that a space is homogeneous if and only if it is doubling: there exists a constant M such that

$$N(X \cap B(x,r), r/2) \leq M$$

for every $x \in X$, $r > 0$.

7. Use the ideas in Lemma 31 and the definition of the Assouad dimension, to prove that if $X - X$ is compact homogeneous subset of a Banach space $\mathcal{B}$ then there exists an $m \in \mathbb{N}$ and a collection (ϕ_n) of elements of $L(\mathcal{B}; \mathbb{R}^m)$ that satisfy $\|\phi_n\| \leq \sqrt{m}$ and

$$z \in X - X \text{ with } \ 2^{-(n+1)} \leq \|z\| \geq 2^{-n} \qquad \Rightarrow \qquad |\phi_n(z)| \geq \frac{1}{8}\|z\|.$$

8. Now use the ideas in Theorem 32 to show that if $X - X$ is a compact homogeneous subset of a Banach space $\mathcal{B}$ then for any $\gamma > 1/2$ there exists a bounded linear map $\Phi\colon \mathcal{B} \to H$, where H is a separable Hilbert space, with

$$\frac{1}{c_L}\frac{\|x-y\|}{|\log\|x-y\||^{\gamma}} \leq |\Phi(x) - \Phi(y)| \leq c_L\|x-y\|.$$