Introduction to Functional Analysis – Errata

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p31 Exercise 2.12: S must also be bounded above (otherwise the supremum need not exist).

p29 The displayed equation in the proof of Proposition 2.29 is incomplete: it should read

$$\sup\{y:\ y \in f(K)\} \in f(K)$$

p37 In the statement of Lemma 3.4, it should be $N\colon X \to [0, \infty)$. Before the final displayed equation in the proof, property (ii) from the statement of the lemma is used ($N(\lambda x) = |\lambda| N(x)$), not property (ii) from Definition 3.1.

p42 The left-hand side of the final displayed equations in the proof of Example 3.13 should be

$$\|\lambda f + (1-\lambda) g\|_{L^p}^p$$

p75 The definitions of g and p in the proof of Corollary 6.3 should be $g(x) := f(a + x(b - a))$ and $p(x) := q((x - a)/(b - a))$.

p85 The space $\mathrm{Lip}_L(X; \mathbb{K})$ and the similar space on p86 are not compact as defined. We need to include a boundedness condition (as required by Corollary 6.13): so, for example, the space

$$\{f \in C(X; \mathbb{K}) : \|f\|_\infty \le B,\ |f(x) - f(y)| \le L|x - y| \text{ for all } x, y \in X\}$$

is compact.

p94ff When defining L^p spaces as completions, a little more care is needed than here. There are two issues: when Ω is unbounded, and when the boundary of Ω has non-zero measure. In the first case (which includes $\mathbb{R}^n$, of course) there may be functions in $C(\overline{\Omega})$ that are not in $L^p(\Omega)$, so here one must take the completion of functions in $C(\overline{\Omega})$ such that $\int_\Omega |f|^p < \infty$. In the second case one has to take the completion of continuous functions with compact support in Ω.

p149 The proof of surjectivity of $T + S$ uses the Contraction Mapping Theorem, so the result guarantees that there is a *unique* x such that $(T+S)x = y$, so $T + S$ is not only surjective, it is also injective. Given a different ordering of the material in the book, one could now appeal to the Inverse Mapping Theorem (Theorem 23.2) to deduce that $(T + S)^{-1}$ is bounded.

p158 Exercise 12.3 (iii) - should read $B(x, x) \geq b\|x\|^2$ for some $b > 0$.

p158 Exercise 12.4 (iii) - should read $f \in H^*$

p170 Third paragraph of proof of Theorem 14.9 is unnecessary: it is an immediate consequence of Proposition 11.18 that $\beta_i I - T$ is invertible for each i, and hence $\beta_i \notin \sigma(T)$.

p178 The spectrum of a bounded operator on a Banach space is also non-empty; but this requires the theory of operator-valued complex functions (see Theorem X in Kreyszig, 1978, for example).

p179 Exercise 15.7: summation in the definition of $S\boldsymbol{x}$ should be over j

p184 After the first sentence of the proof of Theorem 16.6, define λ_1: "set $\lambda_1 = \pm\|T\|$, so that $Tw_1 = \lambda_1 w_1$".

p192 Lemma 17.3. For general functions u_1 and u_2, having zero Wronskian $W_1(u_1, u_2)$ does not imply that u_1 and u_2 are linearly dependent: for example, $u_1(x) = x|x|$ and $u_2(x) = x^2$ have $u_1'(x) = 2|x|$ and $u_2'(x) = 2x$, so $W(u_1, u_2)(x) = 0$ but these functions are linearly independent.

However, if $L[u_1] = L[u_2] = 0$ then zero Wronskian *does* imply linear dependence. To see this, fix any $x_0 \in [a, b]$; then, as written in the current proof, there must exist $\alpha, \beta \in \mathbb{R}$ such that $\alpha u_1(x_0) + \beta u_2(x_0) = 0$ and $\alpha u_1'(x_0) + \beta u_2'(x_0) = 0$. The issue – not addressed in the proof as printed – is that a priori α and β could depend on x. However, if we consider the function $\phi(x) := \alpha u_1(x) + \beta u_2(x)$ then

$$L[\phi] = 0 \qquad \text{with} \qquad \phi(x_0) = \phi'(x_0) = 0.$$

Uniqueness of solutions for this initial-value problem now shows that $\phi(x) \equiv 0$ for all $x \in [a, b]$.

p193 Theorem 17.5: u_1, u_2 here should be in $C^2([a, b])$, not in $\mathcal{D}$. [If they were in $\mathcal{D}$ then Lemma 17.1 would imply that they were both zero.]

p194 The functions u_1 and u_2 constructed in this way satisfy

$$L[u_1] = 0, \quad u_1(a) = 0, \ u_1'(a) = 1 \quad \text{and} \quad L[u_2] = 0, \quad u_2(b) = 0, \ u_2'(b) = 1.$$

They are linearly independent, since otherwise $u_1(a) = u_1(b) = 0$, which coupled with the fact that $L[u_1] = 0$ would imply (by Lemma 17.1) that $u_1 = 0$ (and $u_2 = 0$ too).

p196 That $\mathcal{T}g = u$ follows from the uniqueness of $\mathcal{D}$-valued solutions of $L[u] = g$: if $L[u] = L[v] = g$ then $L[u - v] = 0$, so clearly

$$(L[u - v], u - v) = 0$$

and Lemma 17.1 implies that $u = v$.

p196 The final sentence of the penultimate paragraph should say "in particular, zero is not an eigenvalue of $\mathcal{T}$".

p198 The first case should be when $\lambda < 0$, in which case $\sqrt{\lambda}$ should be replaced by $\sqrt{|\lambda|}$. The final case is then $\lambda > 0$.

p234 line 5 of proof of Lemma 21.6, should read $|\phi(x_n)| \geq n$ since X could be a complex space.

p260 in the displayed equation following (24.1), the denominator should be $\|x_{n_j} - z_{n_j}\|$ (z_{n_j} not y_{n_j}).

p262 Proof of Proposition 24.6: if we set $E_0 = \{0\}$ then it is possible to take $n > m \geq 1$ in the penultimate line.

p294 Theorem 27.12: in the second paragraph of the proof, the separability of Y^* is required in order to invoke Theorem 27.11, which is applied with $X = Y^*$ and $X^* = (Y^*)^* = Y^{**}$.

p297 Exercise 27.10: X here is once again (as in Exercise 27.9) a real Banach space

p366 solution of Exercise 12.3, the coefficient of $B(v, v)$ in first equation after (S.20) should be $\frac{t^2}{2}$.

p369 line -4: ℓ^∞ should be ℓ^2 (twice)

p382 solution of Exercise 20.9: in first line ϕ should read ϕ_n