Introduction to Functional Analysis – Errata

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p24 The first line of the proof of Lemma 2.17 should read ‘for some $\varepsilon > 0$’ (rather than ‘for every $\varepsilon > 0$’).

p31 Exercise 2.12: $S$ must also be bounded above (otherwise the supremum need not exist).

p29 The displayed equation in the proof of Proposition 2.29 is incomplete: it should read

$$\sup \{ y : y \in f(K) \} \in f(K)$$

p37 In the statement of Lemma 3.4, it should be $N : X \to [0, \infty)$. Before the final displayed equation in the proof, property (ii) from the statement of the lemma is used ($N(\lambda x) = |\lambda|N(x)$), not property (ii) from Definition 3.1.

p42 The left-hand side of the final displayed equations in the proof of Example 3.13 should be

$$\| \lambda f + (1 - \lambda)g \|_{L^p}$$

p75 The definitions of $g$ and $p$ in the proof of Corollary 6.3 should be $g(x) := f(a + x(b - a))$ and $p(x) := q((x - a)/(b - a))$.

p85 The space Lip$_L(X; \mathbb{K})$ and the similar space on p86 are not compact as defined. We need to include a boundedness condition (as required by Corollary 6.13): so, for example, the space

$$\{ f \in C(X; \mathbb{K}) : \| f \|_{\infty} \leq B, |f(x) - f(y)| \leq L|x - y| \text{ for all } x, y \in X \}$$

is compact.

p94ff When defining $L^p$ spaces as completions, a little more care is needed than here. There are two issues: when $\Omega$ is unbounded, and when the boundary of $\Omega$ has non-zero measure. In the first case (which includes $\mathbb{R}^n$, of course) there may be functions in $C(\bar{\Omega})$ that are not in $L^p(\Omega)$, so here one must take the completion of functions in $C(\bar{\Omega})$ such that $\int_{\Omega} |f|^p < \infty$. In the second case one has to take the completion of continuous functions with compact support in $\Omega$.

p149 The proof of surjectivity of $T + S$ uses the Contraction Mapping Theorem, so the result guarantees that there is a unique $x$ such that $(T + S)x = y$,
so \( T + S \) is not only surjective, it is also injective. Given a different ordering of the material in the book, one could now appeal to the Inverse Mapping Theorem (Theorem 23.2) to deduce that \((T + S)^{-1}\) is bounded.

p158 Exercise 12.3 (iii) - should read \( B(x, x) \geq b\|x\|^2 \) for some \( b > 0 \).

p158 Exercise 12.4 (iii) - should read \( f \in H^* \).

p170 Third paragraph of proof of Theorem 14.9 is unnecessary: it is an immediate consequence of Proposition 11.18 that \( \beta_i I - T \) is invertible for each \( i \), and hence \( \beta_i \notin \sigma(T) \).

p178 The spectrum of a bounded operator on a Banach space is also non-empty; but this requires the theory of operator-valued complex functions (see Theorem X in Kreyszig, 1978, for example).

p178 Exercise 15.7: summation in the definition of \( Sx \) should be over \( j \).

p184 After the first sentence of the proof of Theorem 16.6, define \( \lambda_1 \): “set \( \lambda_1 = \pm \|T\| \), so that \( Tw = \lambda_1 w \).”

p192 Lemma 17.3. For general functions \( u_1 \) and \( u_2 \), having zero Wronskian \( W_1(u_1, u_2) \) does not imply that \( u_1 \) and \( u_2 \) are linearly dependent: for example, \( u_1(x) = x|x| \) and \( u_2(x) = x^2 \) have \( u_1'(x) = 2|x| \) and \( u_2'(x) = 2x \), so \( W(u_1, u_2)(x) = 0 \) but these functions are linearly independent.

However, if \( L[u_1] = L[u_2] = 0 \) then zero Wronskian does imply linear dependence. To see this, fix any \( x_0 \in [a, b] \); then, as written in the current proof, there must exist \( \alpha, \beta \in \mathbb{R} \) such that \( \alpha u_1(x_0) + \beta u_2(x_0) = 0 \) and \( \alpha u_1'(x_0) + \beta u_2'(x_0) = 0 \). The issue – not addressed in the proof as printed – is that a priori \( \alpha \) and \( \beta \) could depend on \( x \). However, if we consider the function \( \phi(x) := \alpha u_1(x) + \beta u_2(x) \) then

\[
L[\phi] = 0 \quad \text{with} \quad \phi(x_0) = \phi'(x_0) = 0.
\]

Uniqueness of solutions for this initial-value problem now shows that \( \phi(x) \equiv 0 \) for all \( x \in [a, b] \).

p193 Theorem 17.5: \( u_1, u_2 \) here should be in \( C^2([a, b]) \), not in \( D \). [If they were in \( D \) then Lemma 17.1 would imply that they were both zero.]

p194 The functions \( u_1 \) and \( u_2 \) constructed in this way satisfy

\[
L[u_1] = 0, \quad u_1(a) = 0, \quad u_1'(a) = 1 \quad \text{and} \quad L[u_2] = 0, \quad u_2(b) = 0, \quad u_2'(b) = 1.
\]

They are linearly independent, since otherwise \( u_1(a) = u_1(b) = 0 \), which
coupled with the fact that $L[u_1] = 0$ would imply (by Lemma 17.1) that $u_1 = 0$ (and $u_2 = 0$ too).

$p196$ That $Tg = u$ follows from the uniqueness of $\mathcal{D}$-valued solutions of $L[u] = g$: if $L[u] = L[v] = g$ then $L[u - v] = 0$, so clearly

$$(L[u - v], u - v) = 0$$

and Lemma 17.1 implies that $u = v$.

$p196$ The final sentence of the penultimate paragraph should say “in particular, zero is not an eigenvalue of $T$”.

$p198$ The first case should be when $\lambda < 0$, in which case $\sqrt{\lambda}$ should be replaced by $\sqrt{|\lambda|}$. The final case is then $\lambda > 0$.

$p234$ line 5 of proof of Lemma 21.6, should read $|\phi(x_n)| \geq n$ since $X$ could be a complex space.

$p260$ in the displayed equation following (24.1), the denominator should be $\|x_n - z_n\|$ ($z_n$ not $y_n$).

$p262$ Proof of Proposition 24.6: if we set $E_0 = \{0\}$ then it is possible to take $n > m \geq 1$ in the penultimate line.

$p294$ Theorem 27.12: in the second paragraph of the proof, the separability of $Y^*$ is required in order to invoke Theorem 27.11, which is applied with $X = Y^*$ and $X^* = (Y^*)^* = Y^{**}$.

$p297$ Exercise 27.10: $X$ here is once again (as in Exercise 27.9) a real Banach space.

$p301$ “An element $b \in P$ is an upper bound...” ['is' not 'in']

$p366$ solution of Exercise 12.3, the coefficient of $B(v, v)$ in first equation after (S.20) should be $t^2$.

$p369$ line -4: $\ell^\infty$ should be $\ell^2$ (twice)

$p382$ solution of Exercise 20.9: in first line $\phi$ should read $\phi_n$