Comment by “sup” on http://dxdy.ru/topic80156-60.html (translated by S.Chernyshenko)

Re: on Navier-Stokes equation
21.01.2014, 21:43

Заслуженный участник

Появился: 22/11/10
Сообщения: 636

It looks like I constructed a counter-example for the abstract theorem by Otelbaev. The space is l_2. Opertaor A is
$Ae_i = e_i$ for $i < 50$
$Ae_i = ie_i$ for $i \geqslant 50$
Now the bilinear operator L. It is nonzero only in two-dimensional cells
$L(e_{2n}, e_{2n+1}) = 1/n(e_{2n} + e_{2n+1})$, for $n \geqslant 25$
Checking the conditions.
У3. With something to spare: 50.
У2. $(e_i, L(e_i, e_i)) = 0$ for $i \geqslant 50$. For eigenvectors u с $\lambda = 1$ it is also 0, since for them $L(u, u) = 0$
У4. $L(e, u) = 0$ for the eigenvectors e с $\lambda = 1$ also trivially 0.
У1. $(Ax, x) \geqslant (x, x)$ obviously.
Estimate for the operator L
$\|L(u,v)\|^2 = \sum u_{2n}^2 v_{2n+1}^2/n^2 \leqslant C(\sum u_n^2/n)\,(\sum v_n^2/n)$

Now we consider elements $u_n = -n(e_{2n} + e_{2n+1})$. Their norms are obviously increasing. Let $\theta = -1$. Then the negative θ-norms of all these elements are equal to a constant. And $f_n = u_n + L(u_n, u_n) = 0$.

Post by KrgUser on http://dxdy.ru/topic80156-90.html (translated by S.Chernyshenko)

I work in Euro-Asian University, and asked Mukhtarbai Otelbaevich about the counterexample. His reply is below:

The counter-example is correct. The theorem should be corrected by adding the condition: There exists $\delta > 0$ and a family of orthogonal projectors $P_1, P_2, \ldots, P_N$, commuting with the operator A, strongly converging to a unit operator and such that if $\|u + P_N L(P_N u, P_N u)\| \leqslant \delta$, then $\|u\| \leqslant \frac{1}{2}$.
For the parabolic abstract equation and the system of the Navier-Stokes equations (after it is reduced to integral form) this condition is satisfied.

Otelbaev is thankful, and will refer to the post in the English translation of his paper.