

Problem 1. Give an alternative proof of the Gauss-Bournet theorem for translation surfaces that does not assume the existence of a triangulation.

Problem 2. In the slit torus construction we considered only decompositions in irrational directions. Show that if v, w are in rational directions then $|v \times w|$ is bounded below. Show that if s is rational and v, w represent any saddle connection directions then $|v \times w|$ is bounded below.

Problem 3. Consider the enhanced cutting sequence for a rational billiard table. Show that any finite word which appears appears infinitely often with at most k distinct return times where k depends only on the polygon and not on the direction or the word.

Problem 4. Consider a flow in a given direction on a translation surface. According to F.K and Z.K we have a decomposition into surfaces with boundary where each component is minimal or periodic. Show that minimal components have positive genus. This is equivalent to showing that the number of boundary components plus the Euler characteristic is not positive.

Show that having all periodic components is equivalent to having no infinite rays that is every singular trajectory is laterally a saddle connection.

Problem 5. Show that every parabolic automorphism of a translation surface preserves a decomposition into cylinders of rationally related moduli.