

**Course Outline**  
**MA475 Riemann Surfaces**

1. Riemann Surfaces and Atlases
  - (a) Holomorphic functions and the Cauchy-Riemann equations
  - (b) Holomorphic and smooth atlases in arbitrary dimensions
  - (c) Example: The sphere in  $\mathbb{R}^3$  and  $\mathbb{C}\mathbb{P}^1$
  - (d) Holomorphic functions on surfaces
  - (e) The tangent bundle and the chain rule for surfaces
  - (f) Quotient constructions, free and non-free actions
  - (g) Plane curve examples in degree 2
  - (h) Cut and paste construction of the octagon
2. Algebraic Curves (Plane curves)
  - (a) Real and complex plane algebraic curves non-singular examples
  - (b) Construction of  $\mathbb{R}\mathbb{P}^2$  and  $\mathbb{C}\mathbb{P}^2$ , homogeneous polynomials and projective curves
  - (c) Atlases for non-singular algebraic curves and the holomorphic implicit function theorem
3. Holomorphic Maps Between Riemann Surfaces
  - (a) Meromorphic functions on  $\mathbb{C}\mathbb{P}^1$
  - (b) Automorphism groups of the plane, sphere and disk
  - (c) Hyperbolic geometry and the disk
  - (d) The holomorphic inverse function theorem
  - (e) Local form of holomorphic maps
  - (f) Open mapping theorem, non-constant holomorphic functions
  - (g) Proper maps, local homeomorphisms, covering maps and branched covers
  - (h) The degree formula for proper maps
  - (i) The Riemann-Hurwitz Theorem

4. The Weierstrass  $\mathcal{P}$ -function and Elliptic Curves
  - (a) Meromorphic functions on tori of the form  $\mathbb{C}/\Lambda$
  - (b) Using meromorphic functions on tori to parametrise cubic curves
  - (c) Elliptic integrals and holomorphic 1-forms
  - (d) Determining a lattice from a cubic curve
  - (e) Theta functions
  - (f) The moduli space of tori action of  $PSL(2, \mathbb{Z})$
5. Subharmonic functions
  - (a) Simply connected Riemann surfaces and the Uniformization Theorem
  - (b) The definition of harmonic functions in a conformal setting
  - (c) The Poisson integral formula
  - (d) Subharmonic functions (interpretation as charge and potential functions)
  - (e) Green's functions
  - (f) Examples of hyperbolic surfaces
  - (g) The Uniformization Theorem
6. References and Sources
  - (a) "Riemann Surfaces" by Simon Donaldson (Parts I and II)
  - (b) "Complex Algebraic Curves" by Frances Kirwan (Chapters 1-6)  
This book is available electronically through the Warwick Library.
  - (c) "Complex Analysis" by Lars Ahlfors (Chapter 7)
  - (d) Curt McMullen online notes for Math 213a.
  - (e) Curt McMullen online notes for Math 213b. (Pages 1-22) He discusses the regular octagon surface on page 8 and the irregular cover example on page 14.
  - (f) "A primer on Riemann Surfaces" by Alan Beardon. Chapter 9 proves the Uniformization Theorem.
  - (g) "A Scrapbook of Complex Curve Theory" by Herb Clemens
  - (h) "Part IID Riemann Surfaces" by Alexei Kovalev (Cambridge) online.