MA475 Example Sheet 5

7 March, 2019

(Modified 18/3.)

1. Show that the degree of the composition of two proper maps between Riemann surfaces is the product of the degrees.

2. Show that the parallelogram with vertices 0, λ_0, λ_1 and $\lambda_0 + \lambda_1$ has area $|\Im(\lambda_0 \bar{\lambda_1})|$ (where $\Im$ denotes the imaginary part). Show that if λ_0 and λ_1 generate the lattice Λ then this area depends on Λ but not on the choice of generators.

3. Let $\wp$ be the Weierstrass $\wp$-function with respect to a lattice $\Lambda \subset \mathbb{C}$. Say that $\wp(z) = z^{-2} + az^2 + O(z^4)$. Compute the value of a in terms of Λ.

4. Let $\wp$ be the Weierstrass $\wp$-function with respect to a lattice $\Lambda \subset \mathbb{C}$. Show that $\wp$ satisfies the differential equation $\wp''(z) = 6\wp(z)^2 + A$ for some constant A. Show that there are at least three points and at most five points in $\mathbb{C}/\Lambda$ at which $\wp'$ is not locally injective.