

In general if the orbit of the critical point is unbounded similar ideas can be used to show that the dynamics of f_c on \mathbb{C} the Julia set J_c is topologically conjugate to the 1-sided 2 shift. In particular the Julia set is a Cantor set and is totally disconnected.

Example: If c is sufficiently small then f_c is topologically conjugate to the doubling map on the circle. This can be proved by using the ideas of shadowing for expanding maps and using the fact that the Julia set for f_0 is the circle.

It is interesting to look at the parameter space for the family f_c . We color a given pixel corresponding to a c parameter based on the speed at which the critical point goes to ∞ .

c values at which the critical point has a bounded orbit are colored black.

For these c values the julia set J_c is connected. The set of such c values is called the Mandelbrot set.

The largest component of the interior of the Mandelbrot set corresponds to julia sets topologically conjugate to the circle and is called the main cardioid.

It is interesting to compare the parameter space pictures for real and complex quadratic polynomials. The "windows" in the real picture correspond to "islands" in the complex picture.

The "period doubling cascade" in the real picture corresponds to the sequence of islands extending to the left of the main cardioid.

If f_c is uniformly expanding on its Julia set then c lies in an "island" consisting of maps c' which are topologically conjugate to f_c when restricted to the corresponding Julia sets.

It is conjectured that these "hyperbolic islands" are dense in the Mandelbrot set though it has not been proved. It is known that the intersection of the real axis with hyperbolic islands is dense in the real axis.

