MA424 Example Sheet 3

26 October 2015

1. Let f be an orientation reversing circle homeomorphism. We have seen it has exactly two fixed points. Can it have periodic points of other periods? Which periods? Find examples.

2. Let f be a circle homeomorphism with rotation number $\rho(f)$. Find $\rho(f^{-1})$

3. Show that if $g \in C^1([0,1])$ then $\mathrm{var}(g) \leq \max_{x\in[0,1]} |g'(x)|$.

4. Give an example of a compact metric space X, a homeomorphism $f : X \to X$ and a point $x \in X$ such that $\{f^n(x) : n \in \mathbb{Z}\}$ is dense but $\{f^n(x) : n \in \mathbb{N}\}$ is not dense.

   Definition 1 *We say that an interval I in the circle is a* wandering interval *for a circle homeomorphism f if the sets $f^n(I)$ are pairwise disjoint and I is not attracted to a periodic point.*

5. Show that a minimal homeomorphism of the circle does not have any wandering interval.

6. Show that a non-minimal homeomorphism of the circle with irrational rotation number has a wandering interval.