

Analysis I

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Contents

Lecture-by-lecture

— SEQUENCES — WEEK 1

1. Preliminaries. Powers and inequalities. Rationals. 2 has no rational square root.
2. Upper and lower bounds. Least upper bound axiom. $\sqrt{2}$.
3. Archimedean property. Rationals and irrationals are interleaved. Triangle inequality.
+ Dirichlet approximation theorem (online)

— WEEK 2

4. —
5. Sequences and definition of convergence. Examples. Limits are unique. Shift rule.
6. Shift rule. Convergent sequences are bounded. Algebra of limits: examples.

— WEEK 3

7. Algebra of limits 2. Proofs. Limits and inequalities 1.
8. Limits and inequalities 2. Sandwich rule. Examples $(x^{1/n}, n^{1/n})$. Sequences tending to infinity.
9. Powers and factorials. Monotonic sequences 1.

— WEEK 4

10. $(1 + 1/n)^n$ as a monotonic sequence. Decimals.
11. Subsequences and Bolzano–Weierstrass
12. Cauchy sequences and the GPC.

— SERIES — WEEK 5

13. Series and their convergence. Algebra of series. Convergence: $a_n \rightarrow 0$ and vanishing tails.

14. Series with positive terms. Divergence of harmonic series. Comparison test. e is irrational.

15. $(1 + 1/n)^n \rightarrow e$. Absolute convergence. Comparison test 2.

+ The Basel problem

— WEEK 6

16. Ratio test, Root test. Integral test. Convergence of $n^{-\alpha}$, $\alpha > 1$.

17. Bounds on the factorial. Euler's constant. Alternating series test.

18. Rearrangements.

— CONTINUITY — WEEK 7

19. Functions of one real variable. Definition of continuity. Continuity of constants and $x \mapsto \alpha x$.

20. Continuity of x^2 and $1/x$. Preservation of inequality. Algebra of continuous functions (sums and products).

21. Composition of continuous functions. Continuity of quotients. Continuity of $|f|$ and $\max(f, g)$ (statement only, proof on examples). Continuity and sequential continuity are equivalent.

— WEEK 8

22. Discontinuous functions. Sequential continuity. Examples of discontinuity, including Thomae's function.

[23. Open and closed sets and continuity via open sets.]

[24. Geometric definition of sine and cosine. Continuity of trig functions. $x \sin(1/x)$.]

— WEEK 9

25. Intermediate Value Theorem. Applications to roots of equations. All odd polynomials have a real root.

26. Applications of IVT, continued. A fixed point theorem. Definition of an interval. Intervals mapped to intervals. Extreme Value Theorem. Images of closed intervals are

closed.

27. Zeros of even polynomials.

— WEEK 10

28.

29. Uniform continuity.

30. Monotonic functions and their inverses.