

EXERCISES FOR MA4J7 ALGEBRAIC TOPOLOGY II

WEEK 10

- (1) Show that $\mathbb{C}P^2 \vee S^6$ is not homotopy equivalent to any manifold.
- (2) Show that after a suitable change of basis, a skew-symmetric nonsingular bilinear form over \mathbb{Z} can be represented by a matrix consisting of 2×2 blocks $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ along the diagonal and zeros elsewhere. [For the matrix of a bilinear form, the following operation can be realized by a change of basis: Add an integer multiple of the i -th row to the j -th row and add the same integer multiple of the i -th column to the j -th column. Use this to fix up each column in turn. Note that a skew-symmetric matrix must have zeros on the diagonal.]
- (3) If M is an orientable manifold with boundary, show that ∂M is an orientable manifold without boundary.