ERGODIC RAMSEY THEORY – EXERCISES WEEK 5

JOEL MOREIRA

Exercise 6.10. *Show that a system is weak mixing if and only if it is relative weak mixing with respect to the trivial factor (i.e. the factor to the one-point system).*

Exercise 6.12. *Show that an ergodic system is a Kronecker system if and only if it is compact relative to the trivial (one point) factor.*

Exercise 6.13. *Show that the system* $\mathbf{X} = (X, \mathcal{B}, \mu, T)$ *given by* $X = [0,1]^2$*,* $\mathcal{B} = \text{Borel}$*,* $\mu = \text{Lebesgue}$ *and* $T : (x, y) \mapsto (x + \alpha, y + x)$*, where* α *is irrational, is a compact extension of the rotation by* α *(i.e. the system* $\mathbf{Y} = (Y, \mathcal{D}, \nu, S)$ *where* $Y = [0, 1]$*,* $\mathcal{D} = \text{Borel}$*,* $\nu = \text{Lebesgue}$ *and* $S : x \mapsto x + \alpha$*).*

Exercise 6.14. *Let* $\mathbf{X}$ *be as in the previous exercise. Show that the function* $f(x, y) = e(y\lfloor 1/x\rfloor)$ *is not conditionally compact with respect to* $\mathbf{Y}$*.*

Exercise 6.15. *Let* $\mathbf{X}$ *and* $\mathbf{Y}$ *be ergodic systems and let* $\pi : \mathbf{X} \to \mathbf{Y}$ *be a factor map. Let* $(Z, \mathcal{D}, \lambda)$ *be given by Lemma 6.7 and suppose that* Z *is finite. Show that* $\mathbf{X}$ *is a compact extension of* $\mathbf{Y}$*.*

Date: November 21, 2021.