

ERGODIC RAMSEY THEORY – EXERCISES WEEK 7

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- Exercise 8.16.** (1) Show that a set $T \subset \mathbb{N}$ is thick if and only if its complement $\mathbb{N} \setminus T$ is not syndetic.
(2) Show that a set $S \subset \mathbb{N}$ is syndetic if and only if its complement $\mathbb{N} \setminus S$ is not thick.
(3) Show that a set $T \subset \mathbb{N}$ is thick if and only if for any syndetic set $S \subset \mathbb{N}$, the intersection $S \cap T \neq \emptyset$.
(4) Show that a set $S \subset \mathbb{N}$ is syndetic if and only if for any thick set $T \subset \mathbb{N}$, the intersection $S \cap T \neq \emptyset$.

Exercise 8.20. Show that a transitive system (X, T) is minimal if and only if for every non-empty open set $U \subset X$ and every $x \in U$, the set $V(x, U)$ is syndetic.

Exercise 8.24. Show that the point y constructed at the end of the proof of Theorem 8.23 is the indicator function of a syndetic set.

Exercise 8.25. Show that if $y \in \{0, 1\}^{\mathbb{N}_0}$ is the indicator function of a syndetic set then $(0, 0, \dots)$ does not belong to the orbit closure of y under the shift.