Assignment 1

Due Monday 08 October 15:00 (in supervisors' pigeon holes)

1. Is the following statement true for all x and y: "If $x < y$ then $x^2 < y^2$"? What about this statement: "If $x^2 < y^2$ then $x < y$"?

2. (a) Use mathematical induction to prove that if both x and y are positive then $x < y \Rightarrow x^n < y^n$.

(b) Now try to prove the *converse*, that if both x and y are positive then $x^n < y^n \Rightarrow x < y$.

3. Rewrite each of the following expressions without absolute value signs, treating various cases separately where necessary.

$$\text{(a) } a - |(a - |a|)|. \qquad \text{(b) } |(|x| - 2)|.$$

4. Solve the following inequalities:

$$\text{(a) } |x - 1| + |x - 2| \geq 5; \qquad \text{(b) } |x - 1| \cdot |x + 1| > 0.$$

5. Prove the Triangle Inequality. [Hint: Square both sides.]

6. (1) Show, for positive a and b, that $\frac{a+b}{2} - \sqrt{ab} = \frac{(\sqrt{a}-\sqrt{b})^2}{2}$.

(2) Show that the arithmetic mean of two numbers is always greater than or equal to the geometric mean. When can they be equal?