

Assignment 4

Due Monday 29 October 15:00 (in supervisor pigeon hole)

1. Find the limit of each of the sequences defined below

$$(a) a_n = \sqrt[n]{1^2 + 2^2 + \dots + n^2}$$

$$(b) a_n = \frac{n + \sin n^2}{n + \cos n}$$

$$(c) a_n = \frac{1 - 2 + 3 - 4 + \dots + (-2n)}{\sqrt{n^2 + 1}}$$

2. Prove or disprove the following statement

“Suppose $(a_n) \rightarrow a$. If $a_n > 0$ for all n then $a > 0$.”

3. Prove the following theorem:

If $(a_n) \rightarrow a$, $(b_n) \rightarrow b$, and $a_n \leq b_n$ for all n , then $a \leq b$.

4. Let $a_n = n^2$. Write down the first four terms of the three subsequences (a_{n+1}) , (a_{3n-1}) and (a_{2^n}) .

5. Use a calculator to explore the limit of $(2^n + 3^n)^{1/n}$. Now find the limit of the sequence $(x^n + y^n)^{1/n}$ when $0 \leq x \leq y$.

6. Write down the conjectured limit for the power sequence $a_n = x^n$ (Warning: you should get four different possible answers depending on the value of x .) Then prove your conjectures.

7. State whether the following sequences tend to zero or infinity. Prove your answers.

$$(a) \left(\frac{n^{1000}}{2^n} \right) \quad (b) \left(\frac{(1.001)^n}{n} \right) \quad (c) \left(\frac{n!}{n^{1000}} \right) \quad (d) \left(\frac{(n!)^2}{(2n)!} \right)$$

8. Find the limits of the following sequences. As always, show all your work.

$$(a) \left(\frac{n^4 11^n + n^9 9^n}{7^{2n} + 1} \right) \quad (b) \left((4^{10} + 2^n)^{1/n} \right)$$

$$(c) \left(\frac{3n^3 + n \cos^2 n}{n^2 + \sin n} \right) \quad (d) \left((3n^2 + n)^{1/n} \right)$$