

Assignment 7

Due Monday 19 November 15:00 (return to your supervisor's pigeon hole)

Problem 1. Consider the series $1 + 2 + 3 + \dots = \sum_{k=1}^{\infty} k$. State whether each of the following statements is true or false. Do not forget to prove your claims.

(i) $1 + 2 + 3 + \dots = \infty$.

(ii) $1 + 2 + 3 + \dots = -\frac{1}{12}$.

Problem 2. Find the sum of the series

(a) $\sum_{n=1}^{\infty} \frac{1}{10^n}$.

(b) $\sum_{n=1}^{\infty} \frac{9}{10^n}$.

(c) $\sum_{n=1}^{\infty} \frac{1}{4n^2-1}$. Hint: the series $\sum \frac{1}{2n-1}$ and $\sum \frac{1}{2n+1}$ may help.

Problem 3. Find N such that

$$1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{N} > 10$$

Give reasons.

Problem 4. Prove the Sum Rule for series.

Problem 5. Prove the Shift Rule for series.

Problem 6. Give an example of a divergent series $\sum_{n=1}^{\infty} a_n$ for which $(a_n) \rightarrow 0$.

Problem 7. Prove that if $\sum_{n=1}^{\infty} a_n$ converges, then $(a_n) \rightarrow 0$.

Problem 8. Use the Comparison Test to decide whether the following series are convergent or divergent.

(a) $\sum_{n=1}^{\infty} \frac{1}{n^n}$.

(b) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$.

(c) $\sum_{n=1}^{\infty} \frac{n+\sin n}{n^2+\cos n}$.

(d) $d. \sum_{n=1}^{\infty} \frac{n \sin^2 n}{4^n}$.