

Assignment 9

Due Monday 3 December 15:00 (return to your supervisor's pigeon hole)

Problem 1. (i) Let (a_n) be a decreasing null sequence. Show that

$$\left| \sum_{k=N}^{\infty} (-1)^{k+1} a_k \right| \leq a_N.$$

(ii) Let $s = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$. Use the result of (i) to find a value of N so that

$$\left| \sum_{n=1}^N \frac{(-1)^{n+1}}{n} - s \right| \leq 10^{-6}.$$

Problem 2. Find (i) a sequence (a_n) which is non-negative and *strictly* decreasing but where $\sum (-1)^{n+1} a_n$ is divergent; (ii) a sequence (b_n) which is non-negative and null but where $\sum (-1)^{n+1} b_n$ is divergent. In both cases, give reasons.

Problem 3. Using the Alternating Series Test where appropriate, show that each of the following series is convergent:

$$(a) \sum \frac{(-1)^{n+1} n^2}{n^3 + 1}; \quad (b) \sum \frac{2|\cos \frac{n\pi}{2}| + (-1)^n n}{(n+1)^{3/2}}; \quad (c) \sum \frac{1}{n} \sin \frac{n\pi}{2}.$$

Problem 4. Is the series $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\log n}$ absolutely convergent? Convergent?

Problem 5. Is it true: "A series is convergent if and only if it is absolutely convergent"? Explain.

Problem 6. Determine for which values of x the following series converge and diverge. [Make sure you do not neglect those values for which the Ratio Test doesn't apply.]

$$(a) \sum \frac{x^n}{n!}; \quad (b) \sum \frac{n}{x^n}; \quad (c) \sum \frac{(4x)^{3n}}{\sqrt{n+1}}; \quad (d) \sum (-nx)^n.$$

Problem 7. Prove that if (a_n) is a non-negative sequence that tends to a , with $a > 0$, then $\sqrt{a_n} \rightarrow \sqrt{a}$. Prove this by first showing that

$$\sqrt{a_n} - \sqrt{a} = \frac{a_n - a}{\sqrt{a_n} + \sqrt{a}}.$$