MA134 Geometry and Motion

EXAMPLES SHEET 6

Answers to Part B questions must be handed in to your supervisor via the pigeon loft by 2pm Thursday, 21nd February, 2019 (week 7).

Part A. Easier and background questions to be done first. Not to be handed in for marking.

1. Evaluate

$$(a) \quad \int_1^3 \int_0^1 (1+4xy)\, dx\, dy \qquad (b) \quad \int_0^2 \int_0^{\pi/2} x \sin y\, dy\, dx$$

2. Find the volume of the solid lying under the elliptic paraboloid

$$\frac{x^2}{4} + \frac{y^2}{9} + z = 1$$

and above the rectangle $R = [-1,1] \times [-2,2] \times \{z = 0\}$.

3. Evaluate

$$\iint_R y^2\, dA, \qquad R = \{(x,y) | -1 \le y \le 1, -y-2 \le x \le y\}$$

4. Evaluate

$$(a) \quad \int_0^1 \int_0^z \int_0^{x+z} 6xz\, dy\, dx\, dz \qquad (b) \quad \int_0^3 \int_0^1 \int_0^{\sqrt{1-z^2}} ze^y\, dx\, dz\, dy$$

B. Questions for credit

5. Evaluate

$$(a) \quad \iint_R \frac{x^2 y}{y^2+1} dA, \quad R = [-3,3] \times [0,1] \qquad (b) \quad \iint_R xye^{xy^2}\, dA, \quad R = [0,2] \times [0,1]$$

6. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be continuous. Sketch the region and use this to change the order of integration for

$$\int_0^{2a} dx \int_{\sqrt{2ax-x^2}}^{\sqrt{4ax}} dy f(x,y)$$

Here a is a positive number.

7. Let $f : \mathbb{R} \to \mathbb{R}$ be an increasing continuous function. Let $h : \mathbb{R} \to \mathbb{R}$ be its inverse (for any $x \in \mathbb{R}$, $f(h(x)) = x$). Let $g : \mathbb{R}^2 \to \mathbb{R}$ be continuous. Change the order of integration for

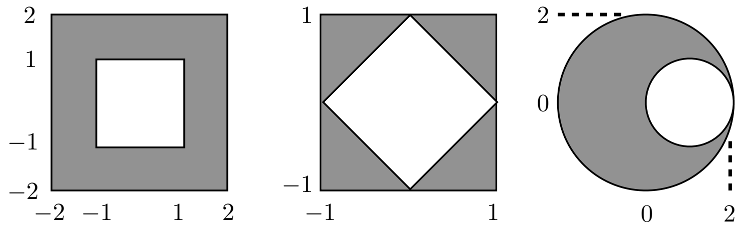
$$\int_a^b dx \int_{f(a)}^{f(x)} dy g(x,y),$$

where $a < b$ are real numbers.

8. Evaluate

$$\int_0^1 dx \int_{\arcsin(x)}^{\pi/2} dy \sin(y)$$

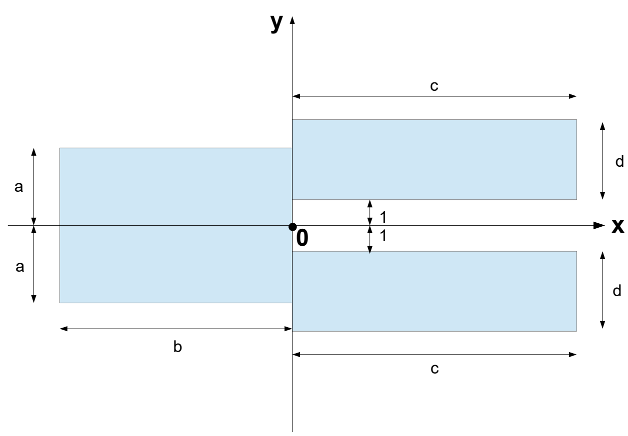
9. Suppose you need to evaluate $\iint_\Omega f\,dA$ over the following shaded domains,



Assume f is integrable everywhere in $\mathbb{R}^2$. In each case, give a formula for $\iint_\Omega f\,dA$ in terms of some repeated integrals with explicit limits of integration. **Remark.** The third domain is bounded by the circles $C_2(0,0)$ and $C_1(1,0)$.

10. Consider the three dimensional solid of constant density ρ bounded by the parabolic cylinder $x = y^2$ and the planes $x = z$, $z = 0$, and $x = 1$. Write down an expression for the total mass as a three dimensional integral. Calculate the resulting integral.

11. Consider the following 'two-dimensional' tuning fork manufactured from a flat piece of thin metal:



Here $b, c, d > 0$ and $a > 1$. Assume that the aerial density (mass per unit square) of the fork is constant. What is c for a perfectly balanced fork, i.e. such that its centre of mass resides at the origin $(0,0)$? Express your answer in terms of a, b, d.

C. More exercises.
(They should not be handed in for marking.)

12. Evaluate by changing the order of integration

$$\int_0^1 \int_{\arcsin y}^{\pi/2} \cos x \sqrt{1+\cos^2 x}\,dx\,dy$$

13. Evaluate

$$\iiint_\Omega x^2 e^y\,dV$$

where Ω is bounded by the parabolic cylinder $z = 1 - y^2$ and the planes $z = 0$, $x = -1$ and $x = 1$.

14. (a) Find the mass of a block in the shape of a unit cube $0 \leq x \leq 1$, $0 \leq y \leq 1$, $0 \leq z \leq 1$, if its density at each point is proportional to (i) the distance from one of the faces and (ii) the square of its distance from one of the vertices. (You can choose which face and which vertex. Your answer will contain a proportionality constant. Call it α.)

    (b) Find the center of mass of the above cube if its density at each point is proportional square of the the distance from the origin.