

MA134 Geometry and Motion Lecture Notes

Lecturer: Oleg Zaboronski

Year: 2018-2019

I	Curves	5
1	Curves and Their Parametrisations	7
1.1	Vector Functions	7
1.2	Curves Defined	8
1.3	Working with Parametrisations	10
	Examples	10
	Sketching	12
	Finding parametrisations	12
1.4	Curves in Multiple Segments	12
1.5	Further Concepts	13
	Basics of vectors	15
	Polar coordinates	16
	Conic sections	16
2	Elementary Calculus of Vector Functions and Curves	17
2.1	Derivative of a Vector Function	17
2.2	Differentiation Rules	18
2.3	A Little Mechanics	18
2.4	Arc Length	19
2.5	Curves in Multiple Segments	21
2.6	Parametrisation by Arc Length	22
	Partitions	24

- Differentiating limits of integrals 24
- Hyperbolic trigonometric functions 24
- Using integral tables 25

- 3 Differential Geometry of Curves 27**
- 3.1 Curves in the Plane 27
 - Unit tangent vector 27
 - Principal normal vector 27
 - Curvature 28
 - Frenet coordinates 29
- 3.2 Curves in Space 30
 - Binormal vector 30
 - Torsion 30
- 3.3 Summary 32
 - General regular parametrisation 32
 - Arc-length parametrisation 32
 - Cross product 34
 - Triple product 35
 - Planes 35

- II Functions of Several Variables 36**

- 4 Differentiation for Functions of Several Variables 37**
- 4.1 Caution on the extension to \mathbb{R}^n 39
- 4.2 Partial Derivatives 40
- 4.3 Gradient 43
- 4.4 Chain Rule 44
- 4.5 Chain Rule (again) 45
- 4.6 Directional Derivative 46
- 4.7 Higher-Order Derivatives 47
 - Quadric surfaces 49
 - Making contour plots 50
 - Optional: linear approximation and the derivative 50

5	Geometry and Applications	53
5.1	Linear Approximation and Tangent to a Graph	53
5.2	Differentials	56
5.3	Level Sets and the Gradient Vector	57
	Contours	57
	Steepest ascent/descent	57
	Extension to functions of three variables	58
5.4	Motion	59
5.5	Critical Points	60
	Recap of curves and surfaces	64
6	Integration: Cartesian Coordinates	67
6.1	Multiple Integration	67
6.2	Iterated Integration over a Rectangle	69
6.3	Multiple Integration in Three or More Variables	71
6.4	Iterated Integration over General Domains	72
6.5	Applications	75
7	Integration: Special Coordinates	77
7.1	Overview	77
7.2	Polar Coordinates	77
7.3	Iterated Integration in Cylindrical Coordinates	79
7.4	Iterated Integration in Spherical Coordinates	81
7.5	Applications	82
III	Functions from \mathbb{R}^n to \mathbb{R}^m	85
8	Functions from \mathbb{R}^n to \mathbb{R}^n	87
8.1	Vector Fields	87
8.2	Coordinate Transformations	88
8.3	Linear Transformations	89
8.4	General transformations in \mathbb{R}^2	92
8.5	Coordinate transformations in \mathbb{R}^3	94
	Why the absolute the value of the Jacobian ?	96

Derivative matrix and the general chain rule	96
9 Surfaces and Surface Integrals	99
9.1 Parametric Surfaces	99
9.2 Tangent Plane and Normal to a Surface	102
9.3 Surface Area	103
9.4 Surface Integrals	105
9.5 Flux Integrals	105
10 Line Integrals	107
10.1 Line Integrals	107
10.2 Line Integrals for Vector Fields	108
10.3 Fundamental Theorem of Line Integrals	109
10.4 Work and Potential Energy	110
10.5 Circulation	111
10.6 Relationship between Various Line and Surface Integrals	112