Joint Kiev-Warwick stochastic analysis seminar

Organisers: Roger Tribe and Oleg Zaboronski

December 11, 2023

Programme

1. **09:45-10:00** Andrey Dorogovtsev, *On the history of Kiev-Warwick collaboration*
2. **10:00-10:45** Ekaterina Glinyanaya, *Gaussian structure in coalescing stochastic flows*
3. **10:50-11:35** Tommaso Rosati, The Allen-Cahn equation with weakly critical initial datum
4. **11:40-12:25** Mykola Vovchanskyi, *Splitting for some classes of non-homeomorphic one-dimensional stochastic flows*
5. **12:25-13:25** Lunch break
6. **13:25-14:10** Guiseppe Cannizzaro, *Weak Coupling Scaling limit of Critical SPDEs*
7. **14:15-15:00** Alexander Weiß, *Intermittency Phenomena for Mass Distributions of Stochastic Flows with Interaction*
8. **15:05-15:50** Roger Tribe, *Coalescing random walks in* $d = 2$
9. **15:55-16:40** Andrey Dorogovtsev, *Equation for knot evolution in random media with folding effect*

Teams meeting ID: 397 149 111 446
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Abstracts

1. **Speaker:** Giuseppe Cannizzaro
   Title: Weak Coupling Scaling limit of Critical SPDEs
   Abstract: The study of stochastic PDEs has known tremendous advances in recent years and, thanks to Hairer's theory of regularity structures and Gubinelli and Perkowski's para-controlled approach, (local) existence and uniqueness of solutions of subcritical SPDEs is by now well-understood. The goal of this talk is to move beyond the aforementioned theories and present novel tools to derive the scaling limit (in the so-called weak coupling scaling) for some stationary SPDEs at the critical dimension. Our techniques are inspired by the resolvent method developed by Landim, Olla, Yau, Varadhan, and many others, in the context of particle systems in the supercritical dimension and might be well-suited to study a much wider class of statistical mechanics models at criticality.

2. **Speaker:** Andrey Dorogovtsev
   Title: Equation for knot evolution in random media with folding effect
   Abstract: Modelling of the evolution of the long linear polymer molecules in the random media is the intriguing problem of the modern probability theory. In spite of the presence a large amount of literature devoted to the static random polymer models and their properties, there are not too much models for moving polymer. In the talk we present a new model for linear polymer evolution. It is a random process in the space of 3D smooth closed curves. Obtained random knot has two essential features. At fixed deterministic moments of the time it has no self-intersections, but changes its topological type at random moments. Also the folding phenomena takes place. Construction is based on the stochastic differential equations with interaction for the stochastic flows. Such equations were introduced by the author for description of evolution of particles in the Euclid space when the motion of every particle depends on the mass distribution of all system. After construction of the moving random knot its topological characteristics are studied. Also the results of computer modelling will be presented.

3. **Speaker:** Ekaterina Glinyanaya
   Title: Gaussian structure in coalescing stochastic flows

Abstract: In this talk, we consider an ordered family of standard Brownian motions on the real line $\{x(u,\cdot),\ u\in\mathbb{R}\}$, in which any two particles move independently until they meet and after that coalesce and move together as one. It is known that for any time $t>0$, the set $x(\mathbb{R},t)$ is almost surely locally finite, so the point measure $N_t([u_1,u_2]) := \#\{x(\mathbb{R},t)\cap[u_1,u_2]\}$ is well-defined. We investigate linear functionals which are represented as integrals with respect to the point process N_t. A limit theorem with respect to the spatial variable is obtained for such integrals. Denote by $X_t^n(f) := \frac{1}{\sqrt{n}}\int_0^n f(u)N_t(du);\ t\in[t_0;T];\ n\geq 1$.

Theorem 1. *For any* $0<t_1<t_2<\ldots<t_m<T$ *the following weak convergence holds* $\left(X_{t_1}^n(f),\ldots,X_{t_m}^n(f)\right)\Rightarrow(\zeta_{t_1}(f),\ldots\zeta_{t_m}(f))$, *as* $n\to\infty$, *where* $(\zeta_{t_1}(f),\ldots\zeta_{t_m}(f))$ *is a Gaussian vector. Moreover, if* f *is a 1-periodic Lipschitz function then the limiting Gaussian process* $\zeta_t(f),t\in[t_0,T]$ $(t_0>0)$ *has continuous modification.*

Considering the family $\{\zeta_f,\ f\in L_2([0,1])\}$ as a generalized Gaussian element, we use it to obtain a limit theorem for the multiple integrals with respect to the point process N_t. Let us denote by $N_t^{(k)}$ the factorial power of the point measure N_t.

Theorem 2. *Let* $f_k:\mathbb{R}^k\to\mathbb{R}$ *be a symmetric function with period 1 w.r.t. all coordinates, such that* $f_k|_{[0;1]^k}\in L_2([0;1]^k)$, $f_k|_{[0;1]^k}=\sum_{i_1,\ldots,i_k}a_{i_1,\ldots,i_k}e_{i_1}\otimes\ldots\otimes e_{i_k}$,
and $\int_0^1 f_k(\vec{x})dx_j=0,\ j=1,\ldots,k$. *Then for even* $k\in\mathbb{N}$

$$\frac{1}{n^{k/2}}\int_0^n \overset{k}{\cdots}\int_0^n f_k(\vec{x})N_t^{(k)}(dx_1\ldots dx_k)\Rightarrow f_k(\zeta_t,\ldots,\zeta_t)+$$
$$+\int_0^1\int_0^1 C_k^2 f_k(\zeta_t,\ldots,\zeta_t,x_{k-1},x_k)G_t(x_{k-1},x_k)dx_kdx_{k-1}+\ldots+$$
$$+\int_0^1\cdots\int_0^1\frac{k!}{(k/2)!2^{k/2}}f_k(x_1,\ldots x_k)G_t(x_1,x_2)\ldots G_t(x_{k-1},x_k)d\vec{x},\ n\to\infty,$$

where

$$f_k(\zeta_t,\ldots,\zeta_t,x_j,\ldots,x_k)=\sum_{i_1,\ldots,i_k=1}^{\infty}a_{i_1,\ldots,i_k}\zeta_t(e_{i_1})*\ldots*\zeta_t(e_{i_{j-1}})\cdot e_{i_j}(x_j)\ldots e_{i_k}(x_k)$$

and $*$ *denotes Wick product of Gaussian random variables.*

4. **Speaker:** Tommaso Rosati
Title: The Allen-Cahn equation with weakly critical initial datum
Abstract: We study the Allen-Cahn equation in dimension 2 with white noise initial datum. In a weak coupling regime, where the non-linearity is damped in relation to the smoothing of the initial condition, we prove Gaussian fluctuations. The effective variance that appears can be described as the solution to an ODE. Our proof builds on a Wild expansion of the solution, which is controlled through precise combinatorial estimates. Joint work with Simon Gabriel and Nikolaos Zygouras.

5. **Speaker:** Roger Tribe
Title: Coalescing random walks in $d = 2$
Abstract: An infinite system of coalescing random walks, with a stationary ergodic initial condition, has a decaying density with asymptotic $Clog(t)/t$ (an old result due to Bramson and Griffeath). In this talk will discuss a proof of this result using a 'modified' rate equation, and its extension to the decay rates for multi-point densities. Joint work with Jamie Lukins and Oleg Zaboronski.

6. **Speaker**: Mykola Vovchanskyi
Title: Splitting for some classes of non-homeomorphic one-dimensional stochastic flows
Abstract: We consider flows of random transformation of the real line that represent an interacting Brownian particle system with common noise and under the action of an external force. The pairwise correlation between particles is assumed to depend on the distance between them. An extreme example of such a system is the celebrated Brownian web in which case particles move independently before a collision happens and merge afterwards.

   The well-known method of operator splitting (known as the Trotter-Kato formula in the theory of semigroups) is applied to these flows so that the actions of the semigroups generated by the flow with zero drift and the ordinary ODE (for drift) are separated.

   Weak convergence of finite-dimensional motions is established. This result is used to derive the convergence of the pushforward measures under the action of the corresponding flows (under some additional assumptions). As another application, the convergence of the associated dual flows in reversed time is obtained. The case of the Brownian web is treated separately.

7. **Speaker:** Alexander Weiß (joint work with Andrey Dorogovtsev)
Title: Intermittency Phenomena for Mass Distributions of Stochastic Flows with Interaction
Abstract: Intermittency is the occurrence of very high but rare peaks in the density, which despite their rarity influence the asymptotic behaviour of the underlying system. Mathematically this can be characterised with the asymptotics of moments. It is an important phenomenon in nature, since it describes the occurrence of extreme events, for example in turbulence theory. In this talk we discuss the intermittency phenomenon for mass distributions of SDEs with interaction:

$$\begin{cases} dx(u,t) &= a(x(u,t),\mu_t)\mathrm{d}t + b(x(u,t),\mu_t)\mathrm{d}B_t \\ x(u,0) &= u \in \mathbb{R}^d \\ \mu_t &= \mu_0 \circ x^{-1}(\cdot,t). \end{cases} \tag{1}$$

Here μ_0 is a probability measure on $\mathbb{R}^d$, in this talk we only consider the case $\mu_0 \ll \mathrm{d}u$. We show the existence of $p_t = \frac{\mathrm{d}\mu_t}{\mathrm{d}u}$ under regularity assumption on the coefficient functions. We furthermore prove the intermittency phenomenon thereof, by investigating the asymptotics of $(p_t)_{t\geq 0}$, under dissipatity conditions on the drift in (1). The talk is based on the investigations in [1].

References

[1] Andrey Dorogovtsev and Alexander Weiß. *Intermittency Phenomena for Mass Distributions of Stochastic Flows with Interaction. 2023.* arXiv: 2304.02571