Title: Partial regularity for a surface growth model.

Date: 1st February 2018,

Abstract: It turns out that the mathematical theory of a 1D surface growth model (SGM),
\[ \partial_t u + u_{xxxx} + \partial_{xx} (u_x)^2 = 0 \]
(which we consider on the torus \( T \)), shares a number of striking similarities to the theory of the 3D incompressible Navier–Stokes equations (NSE). These include local-in-time existence and uniqueness of strong solutions, global-in-time existence of weak solutions, weak strong uniqueness, well-posedness in critical spaces and interior smoothness of strong solutions.

Moreover, the issue of existence of finite-time blow-ups of strong solutions to SGM remains unsolved (similarly as in the case of NSE). However, one can estimate from above the box-counting dimension \( d_B \) of the putative set \( T \) of blow-up times,
\[ d_B(T) \leq 1/4. \]
(The corresponding upper bound in the case of the Navier–Stokes equations is 1/2.)

During the talk we will discuss the recently developed partial regularity theory for SGM (joint work with J. Robinson), which gives sufficient conditions on local (in space-time) behaviour of a weak solution which guarantee local smoothness. As a result, considering the singular set
\[ S := \{ (x,t) : u \text{ is not Hölder continuous in any neighbourhood of } (x,t) \} \]
one can bound its dimension,
\[ d_H(S) \leq 1, \quad d_B(S) \leq 7/6, \]
where \( d_H \) denotes the Hausdorff dimension and \( d_B \) denotes the box-counting dimension.

This theory is an analogue of the partial regularity theory of the Navier–Stokes equations due to Caffarelli, Kohn & Nirenberg (1982). (The corresponding bounds in the case of the Navier–Stokes equations are \( d_H(S) \leq 1, d_B(S) \leq 5/3. \))

Furthermore, we will see that the partial regularity theory for SGM requires certain interior estimates of the biharmonic heat equation (that is \( u_t + u_{xxxx} = 0 \)), as well as the nonlinear parabolic Poincaré inequality,
\[ \| u - [u]_{Q_r} \|_{L^3(Q_r)} \leq C r \left( \| u_x \|_{Q_{2r}} + \| u_x \|_{Q_{2r}}^2 \right), \]
where \([u]_{Q_r} := \frac{1}{|Q_r|} \int_{Q_r} u\), which is valid for any distributional solution of SGM on a cylinder \( Q_{2r} \). Such an inequality is a concept of independent interest; observe that its right-hand side does not involve the time derivative.

Wojciech Ożański, Mathematics Institute, University of Warwick, Coventry CV4 7AL, UK.

E-mail address: W.S.Ozanski@warwick.ac.uk