

Condensed mathematics reading group

Syllabus

The following is a proposed plan for a reading group on condensed mathematics. Overall, it is very open to suggestions and feedback, and we can pivot at any point based on interest. Reference-wise, the proposed primary reference will be the background chapters in [Bri25], supplemented as one finds necessary by the appropriate discussions in [Sch26b],[CS26], [Sch26a]. Other helpful references include [CS20] and [Stu].

Week 1: Introduction, motivations, overview

Discuss the necessity of passing to the condensed setting for reconciling categorical with topological behaviour, having the notion of completion for the sake of $\otimes$ -structures, and homological algebra. With this in hand, give a brief definition of condensed objects, and mention how condensed sets compare to topological spaces. Allude toward solid abelian groups, and how they are generally well-behaved categories in this setting. Dependant on time, add a remark on foundations.

Reference: [Sch26b, Lecture 1].

Week 2: Topological background and main definitions

Overview on the additional point-set background: discuss weights, all relevant notions of κ -light spaces, and characterisations of such spaces. Mention how they correspond to κ -compact objects in appropriate ways. Following that, briefly review Grothendieck topologies as in [Luro9] or [Stu]. Define the pro-étale site of a point, and with that define κ -condensed sets, anima, and κ -condensed categories in general. Give a characterisation of condensed sets as product preserving presheaves (see Example 2.2.3), and condensed anima via condensed functors.

References: [Bri25, Section 2.1, 2.2], [Sch26b, Lecture 1].

Week 3: More perspectives on κ -condensed anima

Build towards Corollary 2.3.7, establishing more models for κ -condensed anima, sketching out the requisite proofs as appropriately as one wants. Mention how this describes κ -condensed sets as appropriate 0-truncated κ -condensed anima. Based on speaker's tastes and background, the focus here should be either on giving some intuition and details about the proofs, or describing the prerequisites used in the proofs of this section.

References: [Bri25, Section 2.3].

Week 4: Condensed objects without cardinals

Introduce the categories of condensed objects in general. Discuss what nice properties carry over (e.g. Giraud's axioms) and what fails (such as accessibility). Describe how these categories are built out of the κ -condensed

ones (Subsection 2.4.1), and then proceed to Subsection 2.4.1 and talk about the embedding of T_1 -topological spaces into condensed sets along with separation/compactness properties of condensed sets.

References: [Bri25, Section 2.4], [Sch26b, Appendix to Lecture 2].

Week 5: Sheaf and condensed cohomology

Define sheaf and condensed cohomology in this setting (preferably as in [Bri25], but the [Sch26a] definitions work equally well). Discuss the comparison between the two, and under what conditions are they isomorphic. Mention some computations of sheaf and condensed cohomology.

References: [Bri25, Section 2.5], [Sch26b, Lecture 3].

Week 6 Condensed module categories

Give a brief introduction, and then go through the nice formal properties discussed in [Bri25, Section 2.6]. Compare the appropriate derived categories. After that, spend some time on module categories over condensed ring spectra (Corollary 2.6.10–2.6.14).

References: [Bri25, Section 2.6], [Sch26b, Lecture 2]

Week 7 Compactness properties of condensed abelian groups

Go over some of the compactness of condensed abelian groups (in regards to cohomology). Introduce the Breen–Deligne resolution ([Sch26b, Appendix to Lecture 4]) and some computations using it.

References: [Bri25, Section 2.7], [Sch26b, Lecture 4]

Week 8: Solid modules part 1

Give definitions and some of the equivalent formulations. Pick a couple of the equivalent ways of describing them and present on that. Discuss the presence of adjoints for the case of κ -solid modules, its formal properties, the derived categories along with the presence of t -structures.

References: [Bri25, Section 2.8], [Sch26b, Lectures 5 and 6].

Week 9: Solid modules part 2

Carry on with the above, and discuss what passes to category of all solid modules. Introduce the symmetric monoidal structure on (κ) -solid modules and some of its formal properties. After that, end on examples and computations.

References: [Bri25, Section 2.8], [Sch26b, Lectures 5 and 6].

For the last week, this is open to suggestions. However, weeks 8 and 9 cover a lot of material, so there is a chance we can also spread this out further throughout the last week (and add the last remaining sections in the chapter). If we've finished all of the above, we can do an intro/overview of the rest of the paper. Alternatively, we can discuss analytic rings, some of the archimedean/complex geometry concepts such as liquid vector spaces as in [CS26], etc.

REFERENCES

- [Bri25] Emma Brink. “Condensed group cohomology”. In: *arXiv preprint arXiv:2512.03648* (2025).
- [CS20] Dustin Clausen and Peter Scholze. *Masterclass in condensed mathematics*. Lecture series. Available online at <https://youtube.com/playlist?list=PLAMniZX5MiiLXPrD4mpZ-O9oiwhev-5Uq&si=PI-3eyBT1PbSuOtc>. 2020.
- [CS26] Dustin Clausen and Peter Scholze. “Condensed mathematics and complex geometry”. In: *arXiv preprint arXiv:2605.11731* (2026).
- [Lur09] Jacob Lurie. “Derived algebraic geometry V: Structured spaces”. In: *arXiv preprint arXiv:0905.0459* (2009).
- [Sch26a] Peter Scholze. “Lectures on analytic geometry”. In: *arXiv preprint arXiv:2605.03655* (2026).
- [Sch26b] Peter Scholze. “Lectures on condensed mathematics”. In: *arXiv preprint arXiv:2605.03658* (2026).
- [Stu] Bernard Le Stum. *An introduction to condensed mathematics*. https://www.bernardlestum.com/_files/ugd/6e3af6_672f5b3efef64fd7b44bba48aae3b77f.pdf.