Introduction to Higher algebraic K-theory - Reading group term 2, 21/22

Talk 1 **Introduction**. Give as motivation the original definitions of K_0 and K_1 for a unital ring (§7). Introduce the necessary preliminaries regarding simplicial sets (including the nerve, geometric realization and classifying space) and bisimplicial sets (p. 64 including $X_L[-], X_R[-]$). Finally state the realization theorem (6.12) and theorem 6.14.

Talk 2 **Waldhausen S-dot construction**. Introduce Waldhausen's S-dot construction (Definition 8.1, 8.2) as well as the bisimplicial version ($S\mathcal{C}[-]$ p. 84, $K[-,-]$ p. 86 and def. 8.5). Extend to exact categories (p.92) by first defining such (def. 8.9, 8.10) as well as exact functors. Finally show that the geometric realization of these are homotopy equivalent (Theorem 8.17) by first proving lemma 8.16.

Talk 3 **Additivity theorem**. Goal is to prove the additivity theorem (9.1), by first constructing i_1, i_2, p_1, p_2 as on page 98, defining $P[-,-]$ (9.5), proving/stating 9.3, 9.4 and 9.5.

Talk 4 **Consequences of the additivity theorem I**. The goal is to prove the cofinality theorem (9.18) as a consequence of the additivity theorem. This is done by first proving 9.15 (simply state lemma 9.10), as well as corollary 9.16 and 9.17.

Talk 5 **Consequences of the additivity theorem II**. The goal is to prove the Resolution theorem (9.20) and Devissage (9.21) as a consequence of the additivity theorem, by first stating Quillen's theorem A (6.5).

Talk 6 **Quillen's Q-construction**. Give the main context of §10. Introduce the Q-construction (p.127) and his version of the higher K-groups (Def 10.1). The goal is then to prove that this agrees with Waldhausen's deinfition (Theorem 10.12). This is done by introducing Segal's subdivision (p.128), constructing the functor $W[-]$ (p.128/129 and definition 10.4), stating/proving parts of 10.6, 10.7, 10.9 and if time permits prove 10.3/10.11.

Talk 7 **Monoidal categories and localization**. Give the basic definitions regarding monoidal categories (11.1 − 11.6) and cofibered functors (11.7, 11.8). Define localization (11.10) and prove proposition 11.11 and 11.12. Finish by giving the definition of quasifibrations and prove Quillen's theorem B for cofibered functors (11.15), using 11.13, 11.14.

Talk 8 **A quasifibration in localization**. The goal is to prove 11.18 and 11.19 using example 11.16 and 11.17.

Talk 9 **K-theory through localization**. The goal is to prove theorem 12.7. Do this by first constructing $\xi(\mathcal{C})$ and $f : \xi(\mathcal{C}) \to Q(\mathcal{C})^{op}$ as on p.155, introduce the action of $i\mathcal{C}$ on $\xi(\mathcal{C})$ as on p.157, prove/state 12.3, 12.4 and 12.6 as well as introduce the homotopy fiber sequence (12.5).

Talk 10 **TBD**.