CHEBOTAREV-TYPE THEOREMS IN HOMOLOGY CLASSES

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Abstract. We describe how closed geodesics lying in a prescribed homology class on a negatively curved manifold split when lifted to a finite cover. This generalizes a result of Zelditch in the case of compact hyperbolic surfaces.

0. Introduction

Given a compact manifold of negative curvature, there are geometric analogues of the Chebotarev Theorem in algebraic number theory due to Sunada [13] (cf. also Parry and Pollicott [8] for the generalization to Axiom A flows). More precisely, given a finite Galois cover of the manifold, these theorems describe the proportion of closed geodesics which lift in a prescribed way to the cover.

In this geometric setting, it is also natural to consider infinite covers and, in particular, the number of closed geodesics lying in a prescribed homology class has been studied by Katsuda and Sunada [4], Phillips and Sarnak [9], Katsuda [3], Lalley [7] and Pollicott [10] (with generalizations to Anosov flows by Katsuda and Sunada [5] and Sharp [12]). In this note we shall combine these points of view, generalizing a result of Zelditch for hyperbolic Riemann surfaces [14].

Let M be a compact smooth Riemannian manifold with negative curvature. Let $\widetilde{M}$ be a finite Galois covering of M with covering group G. For a closed geodesic γ on M, let $l(\gamma)$ denote its length, $\langle\gamma\rangle$ its Frobenius class in G and $[\gamma]$ its homology class in $H = H_1(M, \mathbb{Z})$.

We shall examine how the closed geodesics lying in a fixed homology class $\alpha \in H$, split when lifted to $\widetilde{M}$. More precisely, for a conjugacy class C in G, we study the asymptotics of

$$\pi(T, \alpha, C) = \mathrm{Card}\{\gamma : l(\gamma) \leq T, [\gamma] = \alpha, \langle\gamma\rangle = C\}.$$

The problem is complicated by the fact that that, in general, $[\gamma]$ and $\langle\gamma\rangle$ are not independent quantities. This occurs if the abelian quotient group $G/[G,G]$ is non-trivial, since this group is also a quotient of H, the maximal abelian covering group of M. Let $\pi_G : G \to G/[G,G]$ and $\pi_H : H \to G/[G,G]$ be the natural projections. In particular, the image $\pi_G(C)$ of a conjugacy class $C \subset G$ is a single element in $G/[G,G]$ and if $\pi_G(C) \neq \pi_H(\alpha)$ then $\pi(T, \alpha, C) = 0$, for all values of T.

On the other hand, we have the following result, which extends work of Zelditch for Riemann surfaces [14].

Theorem 1. *If $\pi_G(C) \neq \pi_H(\alpha)$ then $\pi(T,\alpha,C)$ is identically zero.*
If $\pi_G(C) = \pi_H(\alpha)$ then

$$\frac{\pi(T,\alpha,C)}{\pi(T,\alpha)} \to \left|\frac{G}{[G,G]}\right| \frac{|C|}{|G|} \text{ as } T \to +\infty,$$

where $\pi(T,\alpha) = \mathrm{Card}\{\gamma : l(\gamma) \leq T, [\gamma] = \alpha\}$.

Example. Let G be a finite nonabelian nilpotent group, then $[G,G] \neq G$. For definiteness, we can let $G = \{\pm 1, \pm i, \pm j, \pm k\}$ be the Quarternion group of eight elements, then $[G,G] = \pm 1$. Let $\Gamma = \langle a_1, a_2, b_1, b_2 : a_1 b_1 a_1^{-1} b_1^{-1} a_2 b_2 a_2^{-1} b_2^{-1} = 1\rangle$ be a cocompact Fuchsian group. Define a homomorphism $\phi : \Gamma \to G$ by setting $\phi(a_1) = i$, $\phi(a_2) = j$ and $\phi(b_1) = \phi(b_2) = 1$ and extending this to Γ. We can then define a normal subgroup by $\Gamma_0 = \ker(\phi)$. If we set $M = \mathbb{H}^2/\Gamma_0$ and $\widetilde{M} = \mathbb{H}^2/\Gamma_0$ then $\widetilde{M}$ is a finite cover of M with covering group G.

Let us consider a closely related problem. Consider the frame flow $f_t : \mathcal{F}M \to \mathcal{F}M$ on the space of orthonormal frames above M. This is a $SO(n-1)$-extension for the geodesic flow. Changing notation slightly, let γ be a periodic orbit of the geodesic flow, to which we associate a holonomy $\Theta(\gamma) \in SO(n-1)$ which comes from a reference frame being transported around γ. This is defined up to conjugacy. In [8] it was shown that the holonomies were equidistributed on $SO(n-1)$. The following shows that the corresponding result holds for geodesics in a fixed homology class. (Recall that a class function is a function which is constant on conjugacy classes.)

Theorem 2. *Let $F : SO(n-1) \to \mathbb{R}$ be a class function. Then*

$$\frac{1}{\pi(T,\alpha)} \sum_{\substack{l(\gamma) \leq T \\ [\gamma]=\alpha}} F(\Theta(\gamma)) \to \int F d\lambda, \quad \text{as } T \to +\infty,$$

where λ denotes the Haar measure on $SO(n-1)$.

1. Preliminaries

Let M be a compact smooth manifold equipped with a Riemannian metric of negative curvature and let X denote its universal cover. (In the special case where M is a surface with constant curvature -1, X is the hyperbolic plane $\mathbb{H}$.) Then there is a discrete group of isometries $\Gamma \cong \pi_1(M)$ of X such that $M = X/\Gamma$. Now let Γ_0 be a normal subgroup of Γ with finite index. Then $\widetilde{M} = X/\Gamma_0$ is a finite (Galois) covering of M, with covering group $G = \Gamma/\Gamma_0$ (i.e., G acts transitively on the fibres above each point in M).

There is a natural dynamical system, the geodesic flow, associated to M. Let SM denote the unit-tangent bundle of M and, for $(x,v) \in SM$, let $\gamma : \mathbb{R} \to M$ be the unique unit-speed geodesic with $\gamma(0) = x$ and $\dot\gamma(0) = v$. Then the geodesic flow $\phi_t : SM \to SM$ is defined by $\phi_t(x,v) = (\gamma(t), \dot\gamma(t))$ and we shall write h for its topological entropy. There is a one-to-one correspondence between periodic ϕ-orbits and directed closed geodesics on M. The fact that M is negatively curved ensures that the geodesic flow is an Anosov flow and that $h > 0$. This will enable us to use results proved in the context of Anosov flows in this setting.

We shall make use of L-functions defined with respect to certain representations of $\Gamma = \pi_1(M)$. Let $\rho : \Gamma \to U(d)$ be a unitary representation of Γ. We define an L-function $L(s, \rho)$ by the product formula

$$L(s,\rho) = \prod_{\gamma} \det(I - \rho(\langle\langle\gamma\rangle\rangle)e^{-sl(\gamma)})^{-1}, \tag{1.1}$$

where the product is taken over all (prime) closed geodesics γ on M and $\langle\langle\gamma\rangle\rangle$ denotes the homotopy class of γ. Provided $\mathrm{Re}(s)$ is sufficiently large, this will converge to an analytic function. We shall prove Theorem 1 by studying the behaviour of $L(s, \rho)$ for a restricted class of representations.

We shall be interested in how closed geodesics on M lift to $\widetilde{M}$. There are a countable infinity of closed geodesics on M; we shall denote a typical one by γ and its length by $l(\gamma)$. Each such γ has $n = |G|$ lifts $\gamma_1, \dots, \gamma_n$ to $\widetilde{M}$. These lifts are not necessarily closed but, for each $i = 1, \dots, n$, there is a covering transformation $g_i \in G$ relating the endpoints of γ_i and, for $i, j = 1, \dots, n$, g_i and g_j are conjugate. Hence we may associate to γ a well-defined conjugacy class $\langle\gamma\rangle \subset G$, called the Frobenius class of γ. These classes satisfy an analogue to Chebotarev's Theorem in number theory: for a conjugacy class $C \subset G$

$$\lim_{T\to+\infty} \frac{\#\{\gamma : l(\gamma) \le T,\ \langle\gamma\rangle = C\}}{\#\{\gamma : l(\gamma) \le T\}} = \frac{|C|}{|G|}. \tag{1.2}$$

The identity (1.2) is proved by considering L-functions

$$L(s, R_\chi) = \prod_{\gamma} \det(I - R_\chi(\langle\gamma\rangle)e^{-sl(\gamma)})^{-1},$$

where R_χ is an irreducible representation of G with character χ. Since R_χ lifts to a representation of Γ, $L(s, R_\chi)$ is a special case of the L-functions defined by (1.1). The geodesic flow on SM is also covered by the geodesic flow on $S\widetilde{M}$, with covering group G, and hence the analytic properties of $L(s, R_\chi)$ may be deduced from the results in [8].

Lemma 1.1.

(i) *Let* $\mathbf{1}$ *denote the trivial one-dimensional representation of* G. *Then* $L(s,\mathbf{1})$ *is analytic and non-zero on a neighbourhood of* $\{s : \mathrm{Re}(s) \ge h\}$, *apart from a simple pole at* $s = h$.

(ii) *If* $R_\chi \ne \mathbf{1}$ *is an irreducible representation of* G *then* $L(s,\mathbf{1})$ *is analytic and non-zero on a neighbourhood of* $\{s : \mathrm{Re}(s) \ge h\}$.

In this paper, we shall refine (1.1) by requiring that γ lies in a prescribed homology class in $H = H_1(M,\mathbb{Z})$. More precisely, for $\alpha \in H$, we shall write $\pi(T,\alpha) = \#\{\gamma : l(\gamma) \le T,\ [\gamma] = \alpha\}$ and $\pi(T,\alpha,C) = \#\{\gamma : l(\gamma) \le T,\ [\gamma] = \alpha,\ \langle\gamma\rangle = C\}$ and study the ratio

$$\frac{\pi(T,\alpha,C)}{\pi(T,\alpha)}, \tag{1.3}$$

where $[\gamma] \in H_1(M,\mathbb{Z})$ denotes the homology class of γ. Theorem 1 states that either $\pi(T,\alpha,C)$ is identically zero or (1.3) has a limit as $T \to +\infty$. We shall prove this in the next section; however, to do so, we need to first recall how $\pi(T,\alpha)$ behaves as $T \to +\infty$.

The asymptotics of $\pi(T,\alpha)$ are also obtained by considering a family of L-functions, in this case indexed by the characters of H. We suppose that H is infinite and, for simplicity, we consider H modulo torsion. Then these characters may be identified with the torus $\mathbb{T}^d = \mathbb{R}^d/\mathbb{Z}^d$, $d \geq 1$. For $\theta \in \mathbb{T}^d$, we write

$$L(s,\theta) = \prod_{\gamma} \left(1 - e^{-sl(\gamma)+2\pi i\theta\cdot[\gamma]}\right)^{-1}.$$

Since characters of H lift to Γ, this is again an L-function of the form defined in (1.1).

We also write

$$\eta_\alpha(s) = \int_{\mathbb{T}^d} e^{-2\pi i\theta\cdot\alpha} \frac{d^{\nu+1}}{ds^{\nu+1}}(\log L(s,\theta))d\theta,$$

where $\nu = [d/2]$. The following lemma is taken from [5] and [12].

Lemma 1.2. *For each $\alpha \in H$, $\eta_\alpha(s)$ is analytic for* $\mathrm{Re}(s) > h$.

(i) *If d is even then, for some constant $c_0 > 0$,*

$$\lim_{\sigma\to h}\left(\eta_\alpha(\sigma+i\tau) - \frac{(-1)^{\nu+1}c_0}{\sigma+i\tau-h}\right)$$

exists for almost every $\tau \in \mathbb{R}$ and is locally integrable. Moreover, there exists a locally integrable function $f(\tau)$ such that, for $\sigma > h$,

$$\left|\eta_\alpha(\sigma+i\tau) - \frac{(-1)^{\nu+1}c_0}{\sigma+i\tau-h}\right| \leq f(\tau).$$

(ii) *If d is odd then, for some constant $c_0 > 0$,*

$$\lim_{\sigma\to h}\left(\eta_\alpha(\sigma+i\tau) - \frac{(-1)^{\nu+1}c_0\sqrt{\pi}}{\sqrt{\sigma+i\tau-h}}\right)$$

exists for almost every $\tau \in \mathbb{R}$ and is locally integrable with locally integrable first derivative. Moreover, there exists a locally integrable function $f(\tau)$ such that, for $\sigma > h$,

$$\left|\eta_\alpha(\sigma+i\tau) - \frac{(-1)^{\nu+1}c_0\sqrt{\pi}}{\sqrt{\sigma+i\tau-h}}\right| \leq f(\tau).$$

The constant c_0 in (i) and (ii) is independent of α.

When combined with appropriate Tauberian theorems, this Lemma is enough to ensure that, for some constant $c > 0$, independent of $\alpha \in H$,

$$\lim_{T\to+\infty} T^{1+d/2}e^{-hT}\pi(T,\alpha) = c. \tag{1.4}$$

(See [5] or [12] for more details.)

2. Proof of Theorem 1

It is clear that, in general, (1.3) will depend on the relationship between G and H and, particularly, C and α. Write $A = G/[G,G]$, the abelianization of G. Clearly, A is a quotient of $\pi_1(M)$ and, since H is the maximal abelian quotient of $\pi_1(M)$, A is also a quotient of H. The extreme cases are:

(a) G is abelian. Then $G = A$ and G itself is a quotient of H;

(b) G is perfect. Then $G = [G,G]$ and A is trivial.

We shall write $\pi_G : G \to A$ and $\pi_H : H \to A$ to denote the respective projections. In particular, it is clear that if $\pi_G(C) \neq \pi_H(\alpha)$ then $\pi(T,\alpha,C) = 0$ for all $T > 0$.

The proof of Theorem 1 depends on considering L-functions defined with respect to unitary representations of Γ of the form $\theta \otimes R_\chi$, where $\theta \in \mathbb{T}^d$ and R_χ is an irreducible representation of G (or, more precisely, the lifts of these quantities to Γ). However, as we shall describe below, some of these $\theta \otimes R_\chi$ are trivial. The corresponding L-functions take the form

$$\begin{aligned} L(s, \theta \otimes R_\chi) &= \prod_\gamma \det(I - R_\chi(\langle\gamma\rangle) e^{-snl(\gamma) + 2\pi i n \theta[\gamma]}) \\ &= \exp \sum_\gamma \sum_{n=1}^\infty \frac{1}{n} \chi(\langle\gamma\rangle^n) e^{-snl(\gamma)+2\pi i n\theta[\gamma]}, \end{aligned}$$

which converge to analytic functions for $\mathrm{Re}(s) > h$. Taking the logarithm and differentiating $\nu+1$ times gives

$$\left(\frac{d}{ds}\right)^{\nu+1} (\log L)(s, \theta\otimes R_\chi) = \sum_\gamma \sum_{n=1}^\infty n^\nu (-l(\gamma))^{\nu+1} \chi(\langle\gamma\rangle^n) e^{-snl(\gamma)+2\pi i n\theta\cdot[\gamma]}.$$

Applying the standard orthogonality relations for both for irreducible representations of G and for $\mathbb{T}^d$ term-by-term in the above formula we obtain the relation

$$\begin{aligned} &\sum_{R_\chi \text{irred}} \int_{\mathbb{T}^d} e^{-2\pi i\theta\cdot\alpha} \overline{\chi}(C) \left(\frac{d}{ds}\right)^{\nu+1} (\log L)(s,\theta\otimes R_\chi) d\theta \\ &= -\frac{|G|}{|C|} \sum_{\substack{\gamma, n \\ n[\gamma]=\alpha, \langle\gamma^n\rangle = C}} n^\nu l(\gamma)^{\nu+1} e^{-snl(\gamma)} \end{aligned} \tag{2.1}$$

where the Right Hand Side vanishes if $\pi_G(C) \neq \pi_H(\alpha)$.

The asymptotic behaviour of $\pi(T,\alpha,C)$ may be deduced from properties of the Right Hand Side of (2.1) so, to prove Theorem 1, it is enough to study

$$\sum_{R_\chi \text{irred}} \int_{\mathbb{T}^d} e^{-2\pi i\theta\cdot\alpha} \overline{\chi}(C) \left(\frac{d}{ds}\right)^{\nu+1} (\log L)(s,\theta\otimes R_\chi) d\theta \tag{2.2}$$

and understand its meromorphic extension, via that of $L(s,\theta\otimes R_\chi)$, and, in particular, the nature of the singularites on $\mathrm{Re}(s) = h$.

First we determine which $\theta \otimes R_\chi$ are trivial. Let $m = |G/[G,G]|$ be the cardinality of $G/[G,G]$ and let $\mathbf{1} = \chi_0, \cdots, \chi_{m-1}$ be the characters of $G/[G,G]$ (i.e., the 1-dimensional representations of $G/[G,G]$). These lift to G via π_G but 1-dimensional characters on G also descend to $G/[G,G]$, since any such character annihilates commutators. Thus we may identify 1-dimensional representations of G with characters of $G/[G,G]$. Each χ_i also lifts to a character of H, which we can denote by θ_i.

Lemma 2.1. *The representation $\theta \otimes R_\chi$ is trivial precisely when it is of the form $\theta_i^{-1} \otimes R_{\chi_i}$, $i = 0, \ldots, m-1$.*

Proof. It is clear from their construction that these representations are trivial. On the other hand, if $\theta \otimes R_\chi$ is trivial then R_χ is one dimensional and hence that it corresponds to one of the characters $\chi_0, \ldots, \chi_{m-1}$ of $G/[G,G]$. It is easy to see that, for $i = 0, \ldots, m-1$, $\theta \otimes \chi_i$ is trivial only if $\theta = \theta_i^{-1}$.

An immediate consequence is the following.

Lemma 2.2. *For $i = 0, \cdots, m-1$, the function $\zeta(s) := L(s, \theta_i^{-1} \otimes R_{\chi_i})$ has a simple pole at $s = h$ and no other poles on $\mathrm{Re}(s) = h$.*

Next we can consider $L(s, \theta \otimes R_\chi)$ for $\theta \otimes R_\chi$ non-trivial.

Lemma 2.3. *If $L(s, \theta \otimes R_\chi)$ has a pole on $\mathrm{Re}(s) = h$ then R_χ is 1-dimensional and χ is a character of $G/[G,G]$, namely, one of the $\chi_1, \cdots, \chi_{m-1}$.*

Proof. This follows from the discussion on page 146 of [8].

If R_χ is 1-dimensional then, as above, we can lift $\theta \otimes R_\chi$ to $\theta + \theta_i \in \mathbb{T}^{2g}$ and rewrite the L-function as

$$L(s, \theta \otimes R_\chi) = L(s, \theta + \theta_i) = \prod_\gamma (1 - e^{-sl(\gamma) + 2\pi i(\theta+\theta_i)\cdot[\gamma]})^{-1}.$$

However, this is again an L-function for homology.

Lemma 2.4. *If $\theta \otimes R_\chi$ is non-trivial then $L(s, \theta \otimes R_\chi)$ is analytic on $\mathrm{Re}(s) = h$.*

Proof. By the above, we only need to consider the case when $L(s, \theta \otimes R_\chi) = L(s, \theta + \theta_i)$. However, if $\theta \otimes R_\chi$ is non-trivial then $\theta \neq -\theta_i$, so the lemma follows from standard results in [3],[5],[10],[12].

To proceed , we return to the expression (2.1). We can rewrite this as

$$\frac{|C|}{|G|} \sum_{i=0}^{m-1} \overline{\chi_i}(C) \int_{\mathbb{T}^{2g}} e^{-2\pi i \theta\cdot\alpha} \left(\frac{d}{ds}\right)^{g+1} (\log L)(s, \theta \otimes R_{\chi_i}) d\theta + \phi(s)$$

where $\phi(s)$ is a function analytic in a neighbourhood of $\mathrm{Re}(s) = h$ and, from (2.2), $L(s, \theta \otimes R_{\chi_i}) = L(s, \theta + \theta_i)$. We also note that, since we are assuming that $\pi_G(C) = \pi_H(\alpha)$, we have that

$$\overline{\chi}_i(C) e^{-2\pi i(-\theta_i)\cdot\alpha} = 1, \text{ for } i = 0, \cdots, m-1$$

Hence one sees that the function in (2.1) satisfies an analogue of Lemma 1.2 in which c_0 is replaced by $c_0 m|G|/|C|$.

From this one deduces, as in [5] or [12], that

$$\lim_{T\to+\infty} T^{1+d/2} e^{-hT} \pi(T, \alpha, C) = cm\frac{|C|}{|G|},$$

with c as in (1.4).

Finally, recalling that $m = |G/[G,G]|$, this is enough to prove Theorem 1.

Remarks.
(i) If M is either a surface or has curvature which is 1/4-pinched then, working along the lines of [1],[6],[11], one can get a $O(T^{-1})$ error term, as Zelditch obtained for a hyperbolic surface. It is also possible to prove analogous results where a fixed homology class is replaces by one which changes linearly in T (cf. [2], [7]).
(ii) There is a natural extension of Theorem 1 to Anosov flows which are homologically full in the sense of [12], i.e., one for which every homology class is represented by a periodic orbit.

3. Proof of Theorem 2

We can easily adapt the proof of Theorem 1 to prove Theorem 2. Since we are replacing a finite group G by a compact group $SO(n-1)$ we need to consider a countable family of representations R_χ, rather than a finite family. However, by approximation it suffices to consider each representation separately. As in the proof in the last section, one can consider representations $\theta \otimes R_\chi$. However, a significant advantage here is that the groups H and $SO(n-1)$ can be treated independently.

In the case of the trivial representation, we have that $F = \chi = \mathbf{1}$ and we see that

$$\left(\frac{d}{ds}\right)^{\nu+1} (\log L)\,(s, \theta \otimes \mathbf{1})$$

has a singularity of the form

$$Const. \times \frac{1}{(s - s(\theta))}.$$

The analysis reduces to that in [5],[12], from which we get an asymptotic formula

$$\lim_{T \to +\infty} \frac{1}{\pi(T,\alpha)} \sum_{\substack{l(\gamma) \le T \\ [\gamma]=\alpha}} F(\Theta(\gamma)) = 1.$$

However, in the case of non-trivial representations we have that the L-function $L(s, \theta \otimes \mathbf{1})$ is always analytic on $\mathrm{Re}(s) = h$, from which one sees that

$$\sum_{\substack{l(\gamma) \le T \\ [\gamma]=\alpha}} F(\Theta(\gamma)) = o\left(\pi(T,\alpha)\right).$$

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