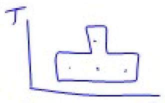
Gibbsian view on space-time phases

The space-time phases for a PCA can be re-defined as the prob. μ on {S-T configs $\underline{\underline{x}}$} s.t. $\forall$ bdd $\Lambda \subset S \times T$ and $\underline{\underline{x}}_{\Lambda^c}$ $(\Lambda^c = S \times T \setminus \Lambda)$, the conditional prob of μ given $\underline{\underline{x}}_{\Lambda^c}$ $\propto$

$$\prod_{\substack{R : R \cap \Lambda \neq \emptyset \\ \subset S \times T}} p_R(\underline{\underline{x}})$$

where R runs over subsets

of $S \times T$ of form



$\{(s,t+1)\} \cup N(s) \times \{t\}$

and $p_R(\underline{\underline{x}}) = p_s(x_s^{t+1} \mid \underline{x}^t)$

Can rewrite $\prod$ as $e^{-\sum_{R : R \cap \Lambda \neq \emptyset} \varphi_R(\underline{\underline{x}})}$

$\varphi_R(\underline{\underline{x}}) = -\log p_R(\underline{\underline{x}})$. This is the defining condition for Gibbs phases in equilibrium stat. mech (where we'd have only S, not $S \times T$) (Dobrushin-Lanford-Ruelle condition) with "energy" fn $\sum_R \varphi_R$

Recall equm stat mech: "$H = \sum_R H_R$"

$\beta \in \mathbb{R}^+$ "coldness" (inverse temperature),

conditional prob for $\underline{x}$ config given $\underline{x}_{\Lambda^c}$

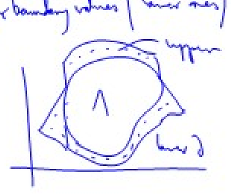
$$= \frac{1}{\underbrace{Z_{\Lambda}(\underline{x}_{\Lambda^c})}_{\text{partition fn}}} e^{-\beta \sum_{R : R \cap \Lambda \neq \emptyset} H_R} = e^{-\beta\left(\sum_{R \cap \Lambda} H_R - F_\Lambda \sum_{s \in \Lambda} 1\right)}$$

$F = \lim_{\substack{|\Lambda| \to \infty \\ \text{van Hove}}} F_\Lambda$ is called free energy per site

e.g. Lebowitz Maes & Speer J Stat Phys ~198?

But note the "energy fns" for PCAs have special feature $F = 0$ because

$$\sum_{\underline{\underline{x}} \text{ on } \Lambda} \prod_{R \cap \Lambda} p_R(\underline{\underline{x}}) = P(\text{upper boundary values} \mid \text{lower ones})$$

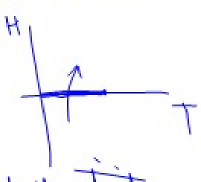


which scales like surface area of Λ, not volume of Λ

So generic ~~statements~~ for ESM don't apply

e.g. Gibbs phase rule which says in ESM generally co-existence of N extremal phases is codim $(N-1)$

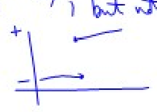
e.g. 2D Ising



Q: Robustness of non-unique phase for PCA?

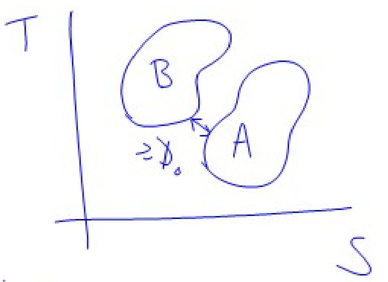
Can prove semi-continuity ("no implosion"), but not continuity

e.g. dyn Ising model "kinetic"



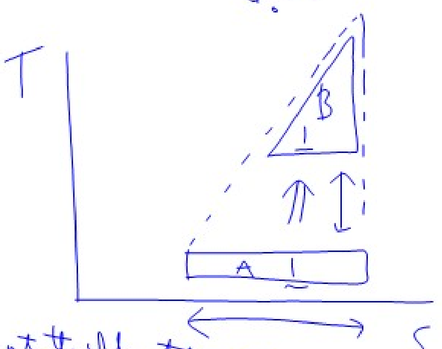
<u>Indecomposability</u> If state-space decomposes into ≥ 2 cpts which do not communicate then get non-unique phase trivially, but don't want to count this as strong emergence, because it suffices to check in which cpt you started.

Say a PCA is <u>indecomposable</u> if $\exists\, D_0 \in \mathbb{R}_+$ s.t. $\forall$ finite subsets $A, B \subset S \times T$ with separation $\geq D_0$ (wrt some metric on $S \times T$) $\forall$ realisations x, x' $\exists$ realisation z agreeing with x on A, x' on B.



cf "specification property in dyn sys"   eg Toom's voter $D_0 = 0$

Stavskaya doesn't satisfy this but deserves to be called strong emergence. finite Stavskaya has a single communicating cpt even though not the whole state space.



So say PCA is <u>pre-indecomposable</u> if it has a unique indecomposable subset and it can be attained in bdd time.

Ch 2. <u>Deterministic Case</u>

1. <u>Examples of coupled map lattices with non-unique S-T phase</u>

CML is a map $f: M \circlearrowleft$ $M = \prod_{s \in S} M_s$

M_s finite-dim mfld e.g. $[-1,+1]$

S countable metric space

We'll turn PCA examples into CML examples.