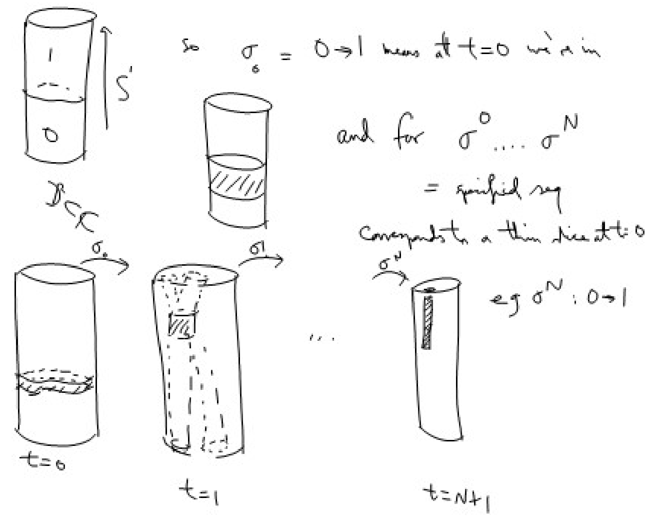
Idea of proof of $\nu f_*^t \to$ some ρ as $t \to \infty$ given by Gibbs potential $\log\left|\det Df|_{E^-}(x^t(\underline{\sigma}))\right|$:

Given abs cts initial prob ν on $B(\Lambda)$ in distant past $t=-M$, what are the relative probs of seeing allowed paths $\sigma^0 \ldots \sigma^N$? (recall σ^t labels transition from symbolic state at time t to $t+1$)

Illustrate for a nonlinear distortion of solenoid map

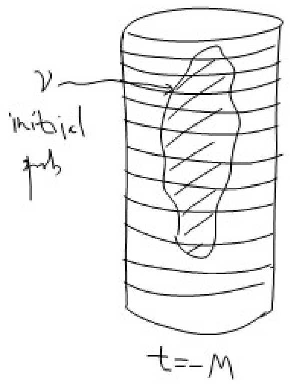


so $\sigma_0 = 0 \to 1$ means at $t=0$ we're in

and for $\sigma^0 \ldots \sigma^N$ = specified seq corresponds to a thin slice at $t=0$

eg $\sigma^N: 0 \to 1$

go back to $t=-M$ but w/o specifying σ^t, $t<0$



2^M thin disks

$P(\sigma^0 \ldots \sigma^N) = \nu$ (set at $t=-M$ giving rise to this seq)

$= \nu(\cup\, 2^M \text{ disks})$

Dependence of $P(\sigma^0 \ldots \sigma^N)$ on seq $\sigma^0 \ldots \sigma^N$ is via thicknesses of these disks

So $P(\sigma^0 \ldots \sigma^N) \underset{\sim}{\propto} \prod \left|\det Df|_{E^-}(x^t)\right|^{-1}$

along associated backwards orbits

More precisely, $\lim_{M\to\infty} P\left(\sigma^0 \ldots \sigma^N \,\middle|\, \sigma^{-M/2} \ldots \sigma^{-1},\ \sigma^{N+1} \ldots \sigma^{N+M/2},\ \nu \text{ at } t=-M\right)$

$$= \frac{1}{Z_{(\text{conditions})\,\bar{\sigma}}} \prod_{t=-\infty}^{+\infty} \frac{\left|\det Df_-(x^t(\bar{\sigma}))\right|}{\left|\det Df_-(x^t(\underline{\sigma}))\right|}$$

where $\bar{\sigma}$ is some particular choice of $\sigma^0 \ldots \sigma^N$ compatible with the chosen condns.

Choose to write this as $\frac{1}{Z} \exp -\sum_t \left(\varphi^t(\underline{\sigma}) - \varphi^t(\underline{\bar{\sigma}})\right)$

with $\varphi^t(\underline{\sigma}) = \log\left|\det Df_-(x^t(\underline{\sigma}))\right|$

φ^t depends exp. weakly on σ^s wrt $|s-t|$, so $\prod$ makes sense & std stat mech (Ruelle) $\Rightarrow \exists!$ prob with these conditionals

③ Uniformly hyperbolic dyns on networks

Setting (S, d) countable metric space

$\forall s \in S$ M_s fin-dim mfld with norm $|\ |_s$ on tgts

$M = \text{``}\prod M_s\text{''}$ with sup norm on tgts

$x^t \in M$ evolves by $x^{t+1} = f(x^t)$

(f is a CML) Suppose $f \in C^1$, in particular

$$\sup_{r \in S} \sum_{s \in S} \left| \frac{\partial f_r}{\partial x_s} \right| < \infty$$

e.g. $x_s' = {}^0f_s(x_s) + \varepsilon \sum_{r \in S} C_{sr}(x_r - x_s)$

$\overset{\shortparallel}{{}^\varepsilon f_s(x)}$ with $\sup_s \sum_r C_{sr} < \infty$, $x_s \in \mathbb{R}$

or could couple cat maps, solenoid maps ...

Orbits of f $\Longleftrightarrow$ fixed pts of $F : M^{\mathbb{Z}} \circlearrowleft$

defined by $\underset{\approx}{x} = (x_s^t) \mapsto {}^\varepsilon f_s(\underset{\sim}{x}^{t-1})$

$\forall (s,t) \in S \times \mathbb{Z}$

Defn: orbit $\underset{\approx}{x}$ is u.hyp if it is a non-degenerate f.p. of F

(using sup-norm on $M^{\mathbb{Z}}$)

Could apply same results as before ($\exists$ splitting, robustness

shadowing, Markov ptn)

except for unique natural prob if $|S| = \infty$

because $M = \prod M_s$ is ∞-dim & then what

is det $Df\_$ in ∞-dimensions?

and would not deduce anything useful abt

dependence on $s \in S$ e.g. spatial corelns of ρ.

So do a space-time version (e.g. Pesin & Sinai)

Assume Df is exponentially local (includes finite

range) i.e. $\exists\ \zeta > 1$, $\varphi : [0, \zeta) \to \mathbb{R}$ s.t.

$$\sup_r \sum_s \left| \frac{\partial f_r}{\partial x_s} \right| z^{d(r,s)} \leq \varphi(z)\ \forall z \in [0, \zeta)$$

Then the u.hyp splitting $TM_x = E^+_{\underset{\approx}{x}} \oplus E^-_{\underset{\approx}{x}}$

is given by exponentially local projections $P^{\pm}_{\underset{\approx}{x}}$

Prove this via Thm: $X = \prod_{s \in S} X_s$, $Y = \prod_{s \in S} Y_s$

with sup norm (appln will be to $S = S \times \mathbb{Z}$) $\boxed{\text{Notation 11 Dec}}$

$L : X \to Y$ linear, φ-exp local, bdd inverse, $y \in Y$

(C, λ)-exp localised around $o \in S$ i.e. $|y_s| \leq C \lambda^{d(s,o)}$

$\Rightarrow$ ...