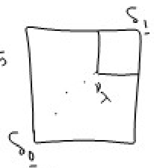
Last time : • $\lim_{t\to\infty} \delta_{\underline{0}} P^t = \nu_\lambda$ exists



• ν_λ non-decr wrt λ

• $\exists\, \lambda_c \in \left(\frac{1}{54}, \frac{1}{2}\right)$ s.t. $\nu_\lambda \begin{cases} = \delta_{\underline{1}} & \forall \lambda \geq \lambda_c \\ < \delta_{\underline{1}} & \forall \lambda < \lambda_c \end{cases}$

Now · If $\lambda < \lambda_c$ then $\nu_\lambda(\underline{1}) = 0$

<u>Proof</u>: else if $\nu_\lambda(\underline{1}) = p > 0$ then conditional measure $\mu = \nu_\lambda(\cdot \mid \text{not } \underline{1})$ is stationary

and $\nu_\lambda = p\delta_{\underline{1}} + (1-p)\mu$

So $\mu < \nu_\lambda$. But $\mu \geq \delta_{\underline{0}}$ so $\mu = \mu P^t \geq \delta_{\underline{0}} P^t \to \nu_\lambda$ $\forall t \geq 0$

So $\mu \geq \nu_\lambda$ ※

• Let $\alpha_A = P\{\text{infection never dies} \mid \text{initial infected set } A\}$

Then $\alpha_A = \nu_\lambda\{\underline{x} : x_s = 0 \text{ iff } s \in A\}$

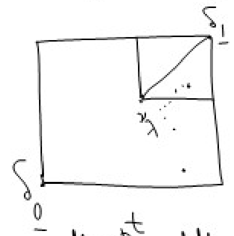
and $\alpha_A = 1$ if A infinite $\left(\alpha_A < 1 \text{ if } A \text{ finite},\ \leq 1 - \lambda^{2|A|}\right)$

(skip proof)

• $\forall \nu \in \mathbb{P}$, $\nu P^t \to \gamma\delta_{\underline{1}} + (1-\gamma)\nu_\lambda$ with

$\gamma = P\{\text{eventual absorption}\}$ $\quad (= 1 - \alpha_A \text{ if } \nu = \delta_A)$

Skip pf (proved for all λ small enough & for all λ large enough) presumably true $\forall \lambda$?)



all νP^t eventually get in a nbhd of the set $\geq \nu_\lambda$

For finite system (on $\mathbb{Z}_N$)

the estimate of $P^0\{x_0^T = 1\}$

does not apply for $T > N$ because fences can wrap round cylinder $\mathbb{Z}_N \times \mathbb{Z}_+$ & then don't have infection stoppers, but can obtain estimates depending on T, N and prove exponentially long time to absorption (wrt N) for λ small

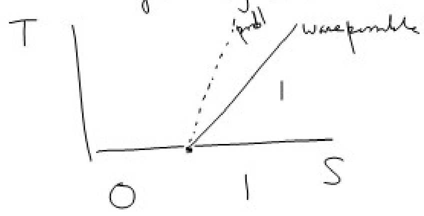
Variations on Stavskaya :

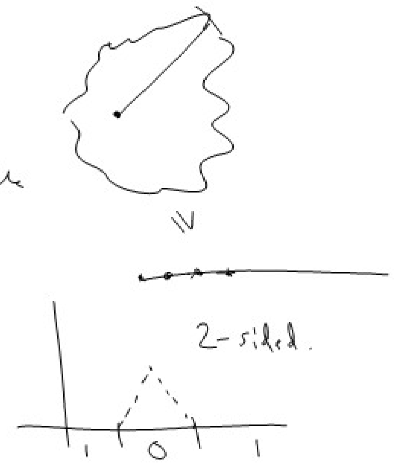
- Can make recovery prob λ_1 & avoiding infection prob λ_2 differ (+ make recovery depend on state of LH nbr) & get "same" results
- Can add infectivity from other side : by monotonicity get 'same' results
- I Don't know about small changes breaking ~~monotonicity~~ (would need to go back to drawing fences)

  (Of course λ large regime is still easy)
- Can go to higher dimensions, infinite networks : see Liggett I for continuous-time (presumably in discrete-time too?)

Proofs compare "radial process" with Stavskaya on half-line, using monotonicity.

- Alternative treatment of Stavskaya by Durrett





2. Toom's majority voter PCA

Stavskaya had feature of an absorbing state

Toom's majority voter has non-unique phase even though communicating. Distinguish from many other voter models (eg Liggett's books) which have absorbing states

Toom NEC : $S = \mathbb{Z}^2$, $x_s \in \{0,1\}$

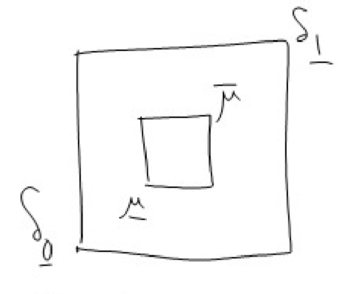
$\cong \{-,+\}$

$$x_s^{t+1} = \begin{cases} \text{majority of } x_s^t,\ x_{s+E}^t,\ x_{s+N}^t & \text{w/prob } 1-\lambda \\ \text{opposite} & \lambda \end{cases}$$

- Exponentially ~~attracting~~ unique sta prob for λ near $\frac{1}{2}$ (because weakly dependent) Ex: find a β s.t. $|\lambda - \frac{1}{2}| < \beta \Rightarrow$ exp erg.

- P is monotone for $\lambda \le \frac{1}{2}$, so $\delta_{\underline{0}} P^t \to$ some $\underline{\mu}$

$\delta_{\underline{1}} P^t \to$ some $\overline{\mu}$



$\underline{\mu} \le \overline{\mu}$ (reflections of each other via $+ \leftrightarrow -$)

- For λ small enough $\underline{\mu} < \overline{\mu}$

idea of proof next time.