What to read up on?

UNIVERSITY OF WARWICK, SUMMER BETWEEN YEAR 1 AND YEAR 2

This is a document that is evolving, as maybe this list is not exhaustive, or the descriptions could be more helpful. If you have any suggestions, please email me at my Warwick email-address:

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and I'll do my best! For now, here are some suggestions for hard, different or complain-worthy topics that come up in second year that you may want to look at/google/read up on, to reduce the culture shock and effort needed to understand them/become fluent with them.

1. TENSOR PRODUCT (OF ABELIAN GROUPS)

Appear in: **Algebra I**, and later.

Uses: quotient groups; presentations.

Given two abelian groups F, G; one can form another group: $F \otimes G$; which is a quotient of $F \times G$.

What to do: get used to calculating with them, understanding elements like "$f_1 \otimes g_1 + f_2 \otimes g_2$" and e.g., $\mathbb{Z}_2 \otimes \mathbb{Z}_3$ as groups.

2. COKERNEL (OF A LINEAR MAP OR HOMOMORPHISM)

Appear in: **Algebra I** (implicitly!) and later.

Uses: quotient groups (or quotient vector space), presentations, commutative diagrams.

The cokernel (like kernel or image) is a group associated to a homomorphism (group analogue of "linear map") which is invariant under "basis changes". This is indispensable in characterising all "finite dimensional" abelian groups, as each can be expressed as the cokernel of some linear map $\mathbb{Z}^n \to \mathbb{Z}^n$, which by standard "basis changes" can be brought into a standard form (this process is applied to a matrix representing the map).

To do: understand the definition, and the invariance hinted at above.

3. METRIC SPACE (A SET WITH STRUCTURE)

Appear in: **Metric Spaces**, **Analysis 3**, **Differentiation** and more.

A metric space is any set in which you have a notion of distance = *metric* between any pair of points. As soon as you do that, you can immediately do all the sequence and continuity stuff in this context, by replacing "distance in $\mathbb{R}$" by "distance", where appropriate.

Most natural, when your space is a vector space, is a *metric* which lines up nicely with your additive structure: this is a *norm*.

To do: understand this definition.

4. Topology (on a set)

Appears in: **Metric Spaces**.

A *topology* is a structure on a set which gives you a notion of “wiggle room”. More precisely, a *topology* tells you what subsets of your space are *open*: intuitively, points in an *open set* have room to wiggle around inside the set (perhaps only an incy wincy bit of room: think “open interval”). This is true when your topology arises from a metric, but the definition of topology extends this in a way that without context just seems unnecessary (like any generalisation, it allows us to do more).

To do: Understand this definition, and also understand what a continuous map between topological spaces is, and perhaps e.g. the definition of ***compact*** *topological space*.

5. Frechét Derivative:

Appears in: **Differentiation**. To do: find the definition (in e.g. the notes) and understand what kind of an object it is

6. A (topological) manifold

Appears in: **Differentiation**.

The Earth appears flat when viewed on a small scale. This makes the surface of the Earth a 2 dimensional manifold. To do: find a definition that makes sense to you.

7. First Isomorphism Theorem, Orbit stabiliser theorem

Appears in: **Algebra 2**

This is all group theory.

Uses: Quotient groups; group actions.

To understand: the definitions of homomorphism, isomorphism (= bijective homomorphism), orbit and stabiliser of a group action.

8. Chinese Remainder Theorem.

Apparently people complain about this. I’ve forgotten what it is.

9. How to write letters: Greek ($\xi, \zeta, \eta, \ldots$) as well as ($fancy/curly\ letters$).

10. Try some Latex (good for a project).