

$$\textcircled{1} X \text{ isotropic} \iff \mathbb{E}[\langle X, x \rangle^2] = \|x\|_2^2 \\ \forall x \in \mathbb{R}^n$$

$$\textcircled{2} X \text{ isotropic} \iff \forall u \in \mathbb{R}^n, \|u\|_2 = 1 \\ \mathbb{E}[\langle X, u \rangle^2] = 1$$

$x \in \mathbb{R}^n \setminus \{0\}$ ($x \equiv 0$ trivial)

$$u = \frac{x}{\|x\|_2}, \text{ then}$$

$$1 = \|u\|_2^2 = \frac{1}{\|x\|_2^2} \mathbb{E}[\langle X, x \rangle^2] \\ = \mathbb{E}[\langle X, u \rangle^2]$$