MA3H2 Term 2

Markov Processes and Percolation theory 15 CATS

Status: List A

Prerequisites: As a prerequisite module students should have done MA359 Measure Theory. [as Probability A is a core module there are no further compulsory prerequisites].

However, it is recommended that students have attended ST213 Mathematics of Random Events. Some basic knowledge of module MA3G7 Functional Analysis I and module ST333 Applied Stochastic Processes may be useful as well.

Commitment: 30 lectures.

Content: This module provides an opportunity to learn the basic mathematics of probability including Markov chains (processes) and percolation with its numerous applications (random growth models (sandpile models), financial mathematics, Markov decision processes, communication networks, operations research).

The module first introduces the mathematics theory of Markov processes (Markov chains with continuous time parameter) running on graphs. An example of a graph is the two-dimensional integer lattice and an example of a Markov process is a random walk on this lattice. A rigorous mathematical introduction to these processes will be given before the module embarks on the discussion of examples. Very interesting problems nowadays involve spatial disorder and dependencies. Therefore, after the main part (from week seven or eight on), an elementary introduction to percolation will be given. Percolation is a simple probabilistic model for spatial disorder, and in physics, chemistry and materials science, percolation concerns the movement and filtering of fluids through porous materials. Recently, percolation theory has brought new understanding and techniques to a broad range of topics in physics, biology, communication networks, materials science as well as geography. Applications include coffee percolation, where the solvent is water, the permeable substance is the coffee grounds, and the soluble constituents are the chemical compounds that give coffee its color, taste, and aroma.

The basic mathematical methods and techniques of random processes and an overview of the most important applications will enable the student to use analytical skills and techniques and models to attack nowadays problems in most professional circumstances (biological and physical systems, communications networks, financial market, operations research, decision processes). The focus is not on the statistics involved in this branch of probability theory but in learning to build up rigorous mathematical models for practical problems. Once having established the mathematical models they can then be attacked and studied by analytical methods and skills.

Leads to: MA482 Stochastic Analysis, MA4F7 Brownian Motion, and MA4H3 Interacting Particle Systems, ST406 Applied Stochastic Processes with Advanced Topics.

Books: We will not follow a particular book. Lecture notes will be available online. Most of our material can be found in the book by J. Norris. A lot of material covered in the module may be found in

O. Häggström, H.O. Georgii, C. Maes: *The random geometry of equilibrium phases*, In: C. Domb and J.L. Lebowitz (eds.) Phase Transitions and Critical Phenomena Vol. 18, Academic Press, London 2000, pp. 1-142. [nicely and clearly written report, however for the module only parts are used]

H.O. Georgii: *Stochastics*, de Gruyter (2008). [basic introduction to stochastics and Markov chains (discrete time)]

G. F. Lawler, L. N. Coyle: *Lectures on Contemporary Probability*, American Mathematical Society (2000) [very nice introductory and easy to read text on Markov chains and random walks]

J. Norris: *Markov chains*, Cambridge University Press [standard reference treating the topic with mathematical rigour and clarity, and emphasising numerous applications to a wide range of subjects]

Y. Suhov, M. Kelbert: *Markov chains: A Primer in Random Processes and their Applications*, Cambridge University Press (2008) [chapter 2 on continuous time Markov chains]

D.W. Stroock: *An Introduction to Markov Processes*, Springer (2005) [basic mathematics on Markov chains in discrete and continuous]

Assessment: 3 hour exam 85%, assignments 15%.

Lecturer: Stefan Adams.