

Status: List C

Commitment: 30 lectures

Prerequisites: ST111/112 Probability A and B, and basic knowledge on Stochastic Processes (e.g. ST202).

Continues: MA3G9 Probability and Discrete Mathematics and ST333 Applied Stochastic Processes.

These modules have related content but are not necessary prerequisites.

Content: This module provides an introduction to basic stochastic models of collective phenomena arising from the interactions of a large number of identical components. We will discuss various examples from physics, biology and social sciences, such as spread of epidemics/opinions, traffic flow, crystal surface growth or population dynamics, and get a qualitative understanding of possible phase transitions. The second main aspect of the course is a proper mathematical description of these models as stochastic processes, and an introduction to basic probabilistic tools for their analysis.

Outline:

- Short introduction to basic theory
Markov processes, graphical construction, semigroups and generators, stationary distributions and reversibility, general remarks on phase transitions, conservation laws and symmetries
- Epidemic models
Contact process, survival and extinction, mean field rate equations, critical values, the DP universality class and percolation; basic tools: coupling, monotonicity
- Random walks with exclusion interaction
exclusion processes, stationary currents and conservation laws, hydrodynamic limits, dynamic phase transition, connection to surface growth and the KPZ universality class
- Random walks with zero-range interaction
zero-range processes and connection to exclusion models, stationary measures and the equivalence of ensembles, phase separation and condensation; basic tools: relative entropy
- Population models
Moran model, Wright-Fisher diffusion and duality with Kingman's coalescent, fixation times, diffusion limits; basic tools: duality

Objectives: At the end of the module students should be acquainted with simple mathematical models of collective phenomena and apply basic probabilistic techniques for their analysis. They should also be familiar with the concept of phase transitions and the role of symmetries/conservation laws.

Books: No single book will be used and detailed lecture notes will be provided.

(Last year's notes are available at <http://www.warwick.ac.uk/~masgav/teaching/ma4h3.html>).

Recommended survey papers for background reading:

- T.M. Liggett: Interacting Particle Systems - An Introduction, ICTP Lecture Notes **17** (2004)
(available online at <http://publications.ictp.it/lns/vol17/vol17toc.html>)
- L. Bertini et al: Stochastic interacting particle systems out of equilibrium, J. Stat. Mech. (2007) P07014
(available online at <http://arxiv.org/abs/0705.1247>)

The following books are cited in the notes and contain the full details on the basic theory if you are interested. However, they are not textbooks and therefore not recommended for introductory reading:

- T.M. Liggett: Interacting Particle Systems, Springer (1985)
- H. Spohn: Large Scale Dynamics of Interacting Particles, Springer (1991)

Organisation: 3h lectures per week;

problem sheets will be provided and solutions discussed in 1h support classes (optional, not every week)

Assessment: 3h exam

Lecturer: Stefan Grosskinsky (S.W.Grosskinsky@warwick.ac.uk)