A note on a stretching filament in Euler flows of $\mathbb{R}^3$

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ABSTRACT

Let $\Gamma(t) \subset \mathbb{R}^3$ be a family of closed, oriented and smooth curves, parametrised by $\alpha(s,t) : S^1 \times [0,T] \to \mathbb{R}^3$ with $\partial_s \alpha = c\ \mathbf{t}$, $c = |\partial_s \alpha|$; here s is the arc-length in S^1 at $t = 0$, and $\mathbf{t}$ is the unit tangent in the Frenet frame $(\mathbf{t}, \mathbf{n}, \mathbf{b})$. Allowing the curve Γ (vortex filament) to vary its length in time, we specify that $\{\Gamma(t)\}_{t\in[0,T]}$ evolves by the curvature (k) - torsion (τ) flow

$$\partial_t \alpha(s,t) = \ k\ \mathbf{b}\ +\ \beta\ \mathbf{n},$$

where β is a free function. We prove that at some time $\mathring{t} \in [0,T]$ if a vorticity concentration condition around $\Gamma(\mathring{t})$ is satisfied, then the velocity of the stretching filament is close to solutions in the Euler equation in a weak sense. The closeness is partly controlled by $\|\tau\|_{L^\infty} \cdot \|k^*\|_{L^{1,\infty}}$ appearing naturally in this geometric setting, k^* being related to the curvature k of Γ. Evolution of a trefoil or torus knots $\Gamma_{p,q}(t)$ by the geometric flow is considered as an example.