New symplectic 4-manifolds and algebraic surfaces

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One of the fundamental problems in the classification of complex surfaces is to find a new family of simply connected surfaces with $p_g = 0$ and $K^2 > 0$. Although a large number of non-simply connected complex surfaces of general type with $p_g = 0$ and $K^2 > 0$ have been known, until recently the only previously known simply connected, minimal, surface of general type with $p_g = 0$ and $K^2 > 0$ was Barlow surface which has $K^2 = 1$. The natural question arises if there are other simply connected surfaces of general type with $p_g = 0$ and $K^2 > 0$ except Barlow surface.

In 2004 I constructed a new simply connected symplectic 4-manifold with $b_2^+ = 1$ and $K^2 = 2$ using a rational blow-down surgery. After this construction, it has been an intriguing question whether such a symplectic 4-manifold admit a complex structure.

In 2006 Yongnam Lee and myself constructed a new simply connected, minimal, complex surface of general type with $p_g = 0$ and $K^2 = 2$ by modifying the symplectic 4-manifold constructed above. Our main techniques used here are a rational blow-down surgery and a $\mathbf{Q}$-Gorenstein smoothing theory.

In 2007 Heesang Park, Dongsoo Shin and myself successfully found a right configuration to construct a simply connected, minimal, complex surface of general type with $p_g = 0$ and $K^2 = 3$ using the same technique as above.

In this talk, I will sketch how to construct such a new family of simply connected symplectic 4-manifolds using a rational blow-down surgery and how to show that such 4-manifolds admit a complex structure using a Q-Gorenstein smoothing theory. If time allows, I will also sketch how to construct a simply connected, minimal, symplectic 4-manifold with $b_2^+ = 1$ (equivalently $p_g = 0$ in complex category) and $K^2 = 4$ using a rational blow-down surgery.

Parts of this talk are based on joint work with Y. Lee or H. Park and D. Shin.