Pricing of First Generation Exotics with the Vanna-Volga Method: Pros and Cons
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Uwe Wystup
Professional

- Diplom in Mathematics, Goethe University Frankfurt
- Ph.D. in Mathematical Finance, Carnegie Mellon University, Pittsburgh
- Professor of Quantitative Finance at Frankfurt School since Oct 2003

- 10 Years of Trading Floor experience at Citibank, UBS, Sal. Oppenheim, Commerzbank as Quant and Structurer
- Expert in FX Options -- Training, Consulting and Software Production for the Financial Industry (MathFinance AG)

Uwe Wystup
Personal

- Married since 1993, two children, one dog
- Lives in Waldems (Taunus)
- Playing for church services in Steinfischbach, Wallrabenstein, Mauloff, Wüstems
- Enjoys Biking, Hiking, Swimming an Golf

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Selected Publications

Jürgen Hakala and Uwe Wystup
Foreign Exchange Risk
Risk Publications, London 2002
http://www.mathfinance.com/FXRiskBook/

Uwe Wystup
FX Options and Structured Products
Wiley Finance, 2006
http://fxoptions.mathfinance.com/

Efficient computation of option price sensitivities using homogeneity and other tricks, joint with Oliver Reiss,

Valuation of exotic options under short selling constraints, joint with Steven E. Shreve and Uwe Schmock,
Finance and Stochastics VI, 2 (2002)

The market price of one-touch options in foreign exchange markets, Derivatives Week Vol. XII, no. 13,
London 2003

Efficient Computation of Option Price Sensitivities for Options of American Style, joint with Christian
Wallner, Wilmott. 2004

The Heston Model and the Smile, joint with Rafal Weron, Chapter contribution to the book Statistical Tools
for Finance and Insurance (STF), eds. Pavel Cizek, Wolfgang Haerdle, Rafal Weron. 2004
Result from the Market: Smile Effect in the Black-Scholes/Merton Model

EUR/USD Smile 14 Feb 2004

Black-Scholes implied vol

10.4 10.9 11.4 11.9 12.4

Put Delta Call

1w 2w 1m 3m 6m 1y 2y

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What are Smile and Skew?

On the axes:

y-axis: implied volatility (in %)

x-axis (for equity)

Strikes

x-axis for FX

Delta of call options (10% and 25%), at-the-money (ATM), Delta of put options (-25% and -10%)
Volatility Smile for Vanilla Options – Model History

- **Black-Scholes: geometric Brownian motion**: \( \sigma \) constant

- **Black-Scholes with time-dependent parameters**

- **Deterministic (local-vol) Model**

- **Stochastic Volatility**

- **Uniform Volatility Model (UVM)**

\[
dS_t = (r_d - r_f) S_t dt + \sigma S_t dW_t
\]
\[
\sigma = \sigma(t)
\]
\[
\sigma = \sigma(S_t, t)
\]

Is the Smile for Vanilla Options deterministic?

- Instantaneous Volatility for the Spot price process depends on:
  - Time to maturity
  - Spot
- and is completely determined by today’s smile
Real Time Volatility Smile Surfaces on Reuters

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<th>45DCALL</th>
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# Implied Volatility in the Options Market

<table>
<thead>
<tr>
<th>EUR/GBP</th>
<th>Spot Rate</th>
<th>Option Volatility</th>
<th>25 Delta Risk Reversal</th>
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<tr>
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<td>0.15 0.16 0.16</td>
</tr>
</tbody>
</table>

Source: BBA (British Bankers’ Association) http://www.bba.org.uk
Butterfly and Risk Reversal

- Risk Reversal: long call + short put
- Butterfly consists of 4 Vanilla Options

OTM put - ATM put - ATM call + OTM call

\[ BF = \frac{1}{2}(\text{OTM call vol} + \text{OTM put vol}) - \text{ATM vol} \]
\[ 0.8 = \frac{1}{2}(9.8 + 10.2) - 9.2 \]

\[ RR = \text{OTM call vol} - \text{OTM put vol} \]
\[ -0.4 = 9.8 - 10.2 \]
Butterfly and Risk Reversal

Smile curve for a fixed expiration time

-25%  ATM  +25%

vol

put delta
call delta

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First Generation Exotics

- **Digitals, Barriers, Touches**
  - KO, KI, RKO, RKI, DKO, DKI
  - OT, NT, DOT, DNT
  - Digitals, EKO, KIKO, TA

- **Compound and Instalments**

- **Asian**
  - Fixed Strike and Floating Strike

- **Others**
  - Lookback
  - Power
  - Chooser
  - Quanto
Barrier Options Terminology

Barrier levels are valid at all times

Call

Regular k.o. (out-of-the-money)

Knock-out barrier

Reverse k.o. (in-the-money)

Payoff at maturity

EUR/USD spot

Strike
Hedging Barrier Options

Vanilla delta is between 0 and 100%

So if we sell a EUR call USD put with delta 40% we need to
buy 40% EUR of the notional
Hedging Barrier Options

Vanilla delta is between 0 and 100%

How about barrier option deltas?

No problem for regular barriers:
Hedging Barrier Options

- Vanilla delta is between 0 and 100%
- How about reverse barrier option deltas?
- They can become arbitrarily large!
How large barrier contracts affect the market

Example: reverse down-and-out put in EUR/USD with strike 1.3000 and barrier 1.2500.
An investment bank delta-hedging a short position with nominal 10 Million has to buy 10 Million times delta EUR.
As the spot goes down to the barrier, delta becomes larger and larger requiring the hedging institution to buy more and more EUR.
This can influence the market since steadily asking for EUR slows down the spot movement towards the barrier and can in extreme cases prevent the spot from crossing the barrier.
Once the barrier is breached, the bank has to unwind the delta hedge, sell lots of EUR -> rate goes down fast.
barrier value function

Buy EUR

Sell lots of EUR

Sell even more EUR

Unwind Hedge = Buy lots of EUR back

Unwind Hedge = Buy lots of EUR back

spot
Hedging Barrier Options

Which volatility should one take to price barrier options?

Or: is there a smile for barrier options?

Answer: not so easy as vol->price is not monotone!

Thus: Given the price

the volatility is not unique

Open question:

Pricing of barrier options

Answer:

look at hedge cost!
Hedging Barrier Options

- E.g. a regular knock-out call with a risk reversal (RR)
- Price of the RR is an indication for the price of the knock-out
- And vanilla smile yields price of the RR
Hedging Barrier Options

- **No KO**: Perfect static hedge
- **KO**: ideally: unwind cost of $RR = 0$. Unfortunately not realistic
- Determine expected unwind cost from experience (e.g. time series analysis / regression of log-returns)
- Add these to the overhedge

---

**Diagram**: 
- Strike of put
- Barrier
- Current spot
- Strike of call
- Spot at maturity

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Pricing RKO Barrier Options

- Compute theoretical value (TV)
- Adjust value based on hedge cost (volatility management)
- Add supplement for delta hedging difficulty
- The latter is approximately the same as: Instead of RKO with barrier B price an RKO with barrier B(1+a), a=1% or a=0.5% depending on risk appetite, details in *Dealing with Dangerous Digitals in Foreign Exchange Risk*
- Discussion of static approaches
Moving towards structuring: Exercise 1

Replicate a double-no-touch using a portfolio of double-knock-out options.

The nominal amounts of the respective double-knock-out options depend on the currency in which the payoff is settled.

In case of a EUR-USD double-no-touch paying 1 USD (domestic currency), determine the nominal amounts of the required double-knock-out calls and puts.

How do you replicate a double-no-touch paying one unit of EUR (foreign currency)?
Moving towards structuring: Exercise 2

Replicate a digital call using vanillas.

How does it work?
What are the problems?
What does this imply for the market price of digitals?
Can we use the same smile volatility for digitals as for vanillas if the strike is the same?
Moving towards structuring: Exercise 3

Replicate European style barriers using digitals and vanillas.

Replicate a KIKO (knock-in-knock-out) with barrier options.
In a KIKO one barrier is a knock-out barrier, the other one a knock-in barrier. Any special concerns?

Replicate a Transatlantic barrier option with vanilla and barrier options. One barrier is of American, the other barrier of European style.
Moving towards structuring: Exercise 4

Replicate reverse knock-out barriers using a portfolio of OT, NT, DOT, DNT, KO, KI, Digitals, Vanillas. You are not allowed to use a RKI.
Problem: a tolerant Double No-Touch Option

Given four barriers A, B, C, D

tolerant double no-touch knocks out after the second barrier is touched or crossed

How to replicate it using barrier and/or touch options?
Accumulators

Consider Fixing Schedule

$N = \text{all Fixings}$

$n = \text{Fixings inside corridor}$

$S_1, S_2, \ldots, S_N$

Accumulative Forward

- Single Accumulation Region
- Double Accumulation Region
- Knock-Out
Accumulator: Pricing/Hedging

- Start with Black-Scholes TV. Compose approximate hedge using first generation exotics, whose overhedge is known.
- E.G. EUR/USD spot 0.9800 of September 24 2002. T=15 months. The client buys a total of 28 million EUR at an improved rate of 0.9150.
- For each EUR/USD fixing between 0.9150 and 1.0500 the client accumulates 28 million EUR divided by the number of fixing days.
- For each EUR/USD fixing below 0.9150 the client accumulates twice this daily amount, such that in the extreme case of all fixing below 0.9150 the total amount accumulated would be 56 million EUR.
- If the non-resurrecting knock out level of 1.0500 is ever traded, then the accumulation stops, but the client keeps 100% of the accumulated amount.
- TV: client receives 400,000 EUR.
Accumulator: Pricing/Hedging – Overhedge Computation

- For the 0.9150 EUR calls RKO at 1.0500 we determine the overhedge as the average of the maturities 6 to 15 months
- We do the same for the 0.9150 EUR puts knock out at 1.0500

<table>
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<th>Tenor</th>
<th>Basis Points (in EUR)</th>
<th>Basis Points (in EUR)</th>
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<tbody>
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<td></td>
<td>RKO calls</td>
<td>KO puts</td>
</tr>
<tr>
<td>6 months</td>
<td>+25</td>
<td>-5</td>
</tr>
<tr>
<td>9 months</td>
<td>+40</td>
<td>-10</td>
</tr>
<tr>
<td>12 months</td>
<td>+40</td>
<td>-20</td>
</tr>
<tr>
<td>15 months</td>
<td>+40</td>
<td>-20</td>
</tr>
<tr>
<td>average</td>
<td>+36</td>
<td>-12</td>
</tr>
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</table>

- On 28 million EUR 36 basis points (bps) are 101,000 EUR, which is the overhedge for buying the RKO calls in the hedge
- Similarly on 56 million EUR 12 bps are 68,000 EUR, which is the overhedge for selling the KO puts. Both are priced at mid market, so fairly aggressive.
Accumulator: Pricing/Hedging – Overhedge Computation

- **cost of knock out**: If the knock out level of 1.0500 is reached, the client has the right to buy the accumulated EUR amount at 0.9150 (with the pre-determined value date), even though the spot is then at 1.0500.

- For the bank selling the accumulative forward this is substantial risk, which can be hedged by buying a 1.0500 one-touch with 15 months maturity.

- The only thing is the notional of this one-touch has to be approximated. First of all the amount at risk, called the parity risk is

  \[ 1.0500 - 0.9150 = 0.1350 \text{ USD per EUR} = 12.86 \% \text{ EUR}. \]

- We approximate the time it takes to reach the parity level of 0.9150 by 7 months and take from the market that the price of a 7 month 0.9150 one-touch is 40%.
Accumulator: Pricing/Hedging – Overhedge Computation

- So say 40% chance below 0.9150, accumulating 22.23 million EUR

- 60% chance above 0.9150, accumulating 16.8 million EUR
- The sum of these two amounts may be the total of 39.03 million EUR accumulated.
- The 15 months 1.0500 one-touch would cost 53.5%
- Parity risk amount equals 39.03 million * 53.5% * 12.86% = 2.7 million EUR
- The one-touch overhedge would be 2.7 million * 3% (mid market) = 81,000 EUR
- To hedge the vega of 206,000 EUR, we take the bid-offer spread of the price in volatilities and arrive at 206,000 * 0.15 vols (bid-offer) = 31,000 EUR.
- \[ \text{Total overhedge} = 101,000 + 68,000 + 81,000 + 31,000 = 281,000 \text{ EUR} \]
Stochastic Volatility

Why stochastic volatility?
Because volatility is stochastic!
E.g.: USD/JPY 1M ATM impl. Vol 1994-2000

Hull/White (1987)
Stein/Stein (1991)
Heston (1993)
Schöbel/Zhu (1998)
Hagan (2000)
...
Heston’s Model

\begin{align*}
\sigma & \quad \text{Instantaneous volatility} \\
\kappa & \quad \text{Mean reversion speed} \\
\theta & \quad \text{Long-term instantaneous variance} \\
\zeta & \quad \text{Volatility of variance (vol of vol)} \\
\rho & \quad \text{Correlation}
\end{align*}

\begin{align*}
\frac{dS_t}{S_t} &= (r_d - r_f) dt + \sigma dW^1_t \\
\sigma &= \sqrt{V} \\
\frac{dV_t}{V_t} &= \kappa (\theta - V_t) dt + \zeta \sqrt{V_t} dW^2_t \\
dW^1_t dW^2_t &= \rho dt
\end{align*}
Heston’s Model

Call value  \( H(t, S, \sigma) \)
Satisfies the PDE

\[
\begin{align*}
H_t + (r_d - r_f)SH_S + \frac{1}{2} S^2 \sigma^2 H_{SS} - r_d H \\
+ \frac{1}{2} \zeta^2 VH_{VV} + \rho \zeta V H_{VS} + [\kappa(\theta - V) - \lambda V] H_V &= 0
\end{align*}
\]

\( \lambda \) Market price of volatility risk, can be set to zero in the calibration
Advantages of Heston’s Model

- Can be implemented
- Covers wide product range
- Explains market prices
Case Study: One-touch with Heston vs Vanna-Volga

- Pays a fixed amount of a pre-specified currency (EUR or USD), if EUR/USD touches or crosses a barrier at any time up to expiration
- Price is between 0% and 100% of the notional
- The closer the spot to the barrier, the higher the price of the one-touch
- Notional is paid at maturity (standard) or at first hitting time
- Price is often far from theoretical value, why?
- Cost of risk managing the volatility exposure
One-touch

- Price is often far from theoretical value (TV), why?
- Cost of risk managing the volatility exposure (Overhedge)
- Examples with lower and upper barrier

Market data: EUR/USD 17 July 2002 1.0045 EUR 3.33% USD 1.76%, 3 M ATM vol 11.85%, RR 1.25%, BF 0.25%
How does vanna-volga work?

Market price of an exotic Option

\[ TV = \]  

\[ + p \cdot \text{vanna of the Option} \cdot \text{OH RR} / \text{vanna RR} \]  

\[ + p \cdot \text{volga of the Option} \cdot \text{OH BF} / \text{volga BF} \]  

RR: Risk Reversal  
BF: Butterfly  
p: Probability that the hedge is needed  
OH: overhedge = market price – Black-Scholes TV
Volatility Risk for Options

- Goal: Compute the cost of vega management
- Vanna = change of vega when spot changes
- → compute the cost of vanna
- Volga = change of vega when volatility changes
- → compute the cost of volga
- Overhedge to Black-Scholes TV = total cost of vanna and volga
vanilla

time to expiration (days)
Vanna-Volga Literature

- Patent file of SuperDerivatives (google!)
How does vanna-volga work?

- Example: USD/JPY 1-year one-touch at barrier 127.00 with Nominal in USD
- Market data: spot 117.00, Volatility 8.80%, USD rate 2.10%, JPY rate 0.10%, 25Delta RR -0.45%, 25 Delta BF 0.37%
- TV: 38.2%
- Vanna: -9.0
- Volga: -1.0
How does market-oriented valuation work?

Market price of an exotic Option
= TV
+ p • vanna of the Option • OH RR / vanna RR
+ p • volga of the Option • OH BF / volga BF

Example USD/JPY one-touch
= 38.2%
+ p • [-9.0 • (-0.15%) / 4.5]
+ p • [-1.0 • 0.27% / 0.035 ]
= 38.2% + p • [0.3% - 7.7%]
= 38.2% - p • 7.4%

Hitting probability: 38.2%
Hedge is not needed with 38.2% probability
Thus, p = 100% - 38.2% = 61.8%
Total overhedge: 61.8% • -7.4% = -4.6%
Market price: 38.2% - 4.6% = 33.6%
Bid/Ask: 32% / 35%
Hedge of a long position: sell 2 RR and 28 BF

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Heston vs. Market for the One-Touch

Price supplement for 6m USD/JPY one-touch options

-5% -4% -3% -2% -1% 0% 1% 2% 3% 4%

0% 10% 20% 30% 40% 50% 60% 70% 80% 90% 100%

TV

supplement

Heston
Market

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Is Vanna Volga Pricing consistent with the Smile?
Is Vanna Volga Pricing consistent with the Smile?
Is Vanna Volga Pricing consistent with the Smile?
What to take for \( p \)?

<table>
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<th>Product</th>
<th>( p )</th>
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<td>KO, RKO, DKO</td>
<td>No-Touch probability</td>
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<tr>
<td>DNT</td>
<td>0.5</td>
</tr>
<tr>
<td>OT</td>
<td>( 0.9 \times \text{No-Touch probability} - 0.5 \times \text{bid-ask-spread} \times (TV - 33%) / 66% )</td>
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## First Generation Exotics

<table>
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<tr>
<th>Product</th>
<th>Valuation via</th>
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<td>Digital options</td>
<td>Vanilla Spread</td>
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Example: Reverse Knock Out EUR call USD put

- $T=182$ days
- Strike 1.5000
- $AMT=10kEUR$
- Spot 1.5500
- Feb 28 2008
- USD=2.99%
- EUR=4.43%
- ATM FWD=8.64%

![Smile Chart](image)
Example: Reverse Knock Out EUR call USD put

<table>
<thead>
<tr>
<th>RKO TV</th>
<th>Overhedge</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>-120</td>
</tr>
<tr>
<td>8</td>
<td>-100</td>
</tr>
<tr>
<td>12</td>
<td>-80</td>
</tr>
<tr>
<td>16</td>
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<tr>
<td>24</td>
<td>-20</td>
</tr>
<tr>
<td>28</td>
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<tr>
<td>32</td>
<td>20</td>
</tr>
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<td>36</td>
<td>40</td>
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<tr>
<td>40</td>
<td>60</td>
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<tr>
<td>44</td>
<td>80</td>
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<tr>
<td>48</td>
<td>100</td>
</tr>
<tr>
<td>52</td>
<td>120</td>
</tr>
<tr>
<td>56</td>
<td>140</td>
</tr>
</tbody>
</table>

- Vanna-Volg Overhedge
- Delta Overhedge
- Total Overhedge

E.g. Barrier 1.6500, TV 215, OH(vv) 16
E.g. Barrier 1.6500, TV 215, OH(vv) 16

Mid Market 231, OH(delta) 28
Basket Call: Case Study

Scenario: USD, GBP and JPY to be changed into EUR

Market volatilities (from Reuters) are

<table>
<thead>
<tr>
<th>Market data</th>
<th>2-Jul-04</th>
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</thead>
<tbody>
<tr>
<td>volatilities</td>
<td>FX pair</td>
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<tr>
<td>9.00</td>
<td>GBP/USD</td>
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<tr>
<td>9.90</td>
<td>USD/JPY</td>
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<tr>
<td>10.40</td>
<td>GBP/JPY</td>
</tr>
<tr>
<td>10.10</td>
<td>EUR/USD</td>
</tr>
<tr>
<td>7.40</td>
<td>EUR/GBP</td>
</tr>
<tr>
<td>10.30</td>
<td>EUR/JPY</td>
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</tbody>
</table>
Basket Call: Case Study

**Scenario:** USD, GBP and JPY to be changed into EUR

**Maturity:** 3 months

**FX volatilities imply the correlations**

<table>
<thead>
<tr>
<th>correlation</th>
<th>spot</th>
<th>1.2150</th>
<th>0.6690</th>
<th>132.50</th>
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<tbody>
<tr>
<td>GBP/USD</td>
<td>USD/JPY</td>
<td>GBP/JPY</td>
<td>EUR/USD</td>
<td>EUR/GBP</td>
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<tr>
<td>1.00</td>
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<td>0.49</td>
<td>0.71</td>
<td>-0.25</td>
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<tr>
<td>-0.40</td>
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<td>0.61</td>
<td>-0.47</td>
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<tr>
<td>0.49</td>
<td>0.61</td>
<td>1.00</td>
<td>0.16</td>
<td>-0.37</td>
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<td>0.71</td>
<td>-0.47</td>
<td>0.16</td>
<td>1.00</td>
<td>0.51</td>
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<tr>
<td>-0.25</td>
<td>-0.16</td>
<td>-0.37</td>
<td>0.51</td>
<td>1.00</td>
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<tr>
<td>0.31</td>
<td>0.50</td>
<td>0.74</td>
<td>0.53</td>
<td>0.35</td>
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</table>
Basket Call: Case Study

**Scenario:** USD, GBP and JPY to be changed into EUR

**Comparing a basket put with 3 single vanilla puts**

<table>
<thead>
<tr>
<th>Base Currency</th>
<th>EUR</th>
<th>Basket Put EUR call</th>
<th>rate</th>
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<tbody>
<tr>
<td>notional</td>
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<tr>
<td>currencies</td>
<td>USD</td>
<td>JPY</td>
<td>GBP</td>
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<tr>
<td>weights</td>
<td>33.33%</td>
<td>33.33%</td>
<td>33.33%</td>
</tr>
<tr>
<td>Spot</td>
<td>1.2150</td>
<td>132.50</td>
<td>0.6690</td>
</tr>
<tr>
<td>1/Sport</td>
<td>0.8230</td>
<td>0.0075</td>
<td>1.4948</td>
</tr>
<tr>
<td>Strikes in EUR</td>
<td>1.2195</td>
<td>133.33</td>
<td>0.6667</td>
</tr>
<tr>
<td>volatilities in %</td>
<td>10.10</td>
<td>10.30</td>
<td>7.40</td>
</tr>
<tr>
<td>interest rates in %</td>
<td>1.61</td>
<td>-0.04</td>
<td>4.86</td>
</tr>
<tr>
<td>sum</td>
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<tr>
<td>Vanilla Prices</td>
<td>EUR</td>
<td>178,000.00</td>
<td>60,000.00</td>
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<tr>
<td>Basket Price</td>
<td>EUR</td>
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<tr>
<td>Save</td>
<td>EUR</td>
<td>38,000.00</td>
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</tbody>
</table>
Basket Call: Premium Saved

Basket option vs. two vanilla options

Two vanilla calls

Premium saved

Basket call

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Basket Options


Market price (quick): quote 10 basis points wider

Market price (more precise): compare with a portfolio of vanillas with strikes chosen such that the basket is approximated as good as possible. From these vanillas derive a) the overhedge and b) a hedge. Details on Formula Catalogue of http://www.mathfinance.com under basket
Model for a Multi-Currency Market

- **N-dimensional geometric Brownian motion**

- **(1) GBP/USD**

- **(2) USD/JPY**

- **(3) GBP/JPY**

- **W**: standard Brownian motion

- **μ**: Drift (from rates)

- **σ**: Volatilities

- **ρ**: Correlations

\[
\begin{align*}
    dS_t^{(i)} &= \mu_i S_t^{(i)} dt + \sigma_i S_t^{(i)} dW_t^i \\
    dW_t^i dW_t^j &= \rho_{ij} dt \\
    i &= 1, \ldots, N
\end{align*}
\]
For Option Valuation we need

- Interest rates
- Volatilities
- Correlation coefficients

Correlation is not quoted, not traded, not observable

Solution: Use the dependence of FX spots
Triangular Relationship

1. GBP/USD
2. USD/JPY
3. GBP/JPY

\[ S_t^{(1)} \cdot S_t^{(2)} = S_t^{(3)} \]

\[ \Rightarrow \text{var} \log S_t^{(1)} + \text{var} \log S_t^{(2)} + 2 \text{cov}(\log S_t^{(1)}, \log S_t^{(2)}) = \text{var} \log S_t^{(3)} \]

\[ \Rightarrow \sigma_1^2 + \sigma_2^2 + 2\sigma_1\sigma_2\rho_{12} = \sigma_3^2 \]

\[ \Rightarrow \rho_{12} = \frac{\sigma_3^2 - \sigma_1^2 - \sigma_2^2}{2\sigma_1\sigma_2} \]
Triangular Relationship: Geometric Interpretation

\[ S_t^{(1)} \cdot S_t^{(2)} = S_t^{(3)} \]

\[- \rho_{12} = \cos \phi_{12} \]

\[ \sigma_1^2 + \sigma_2^2 - 2\sigma_1 \sigma_2 \cos \phi_{12} = \sigma_3^2 \]

Law of Cosine
Extensions

- What is the correlation between USD/JPY and GBP/EUR?
- Does the law of cosine work in higher dimensions like in a tetrahedron?
- What else do we need?
- Does the method extend to equity options?
FX Market in a Tetrahedron

We need 15 correlation coefficients

12=3\times4 from triangular markets

The remaining 3 are

\[ \rho_{16}, \rho_{25}, \rho_{34} \]
FX Market in a Tetrahedron

**Example**

\[ \rho_{34} \]

\[ S_i^{(4)} \cdot S_i^{(2)} = S_i^{(6)} \]

\[ \Rightarrow \text{cov}(\log S_i^{(3)}, \log S_i^{(4)}) = \text{cov}(\log S_i^{(3)}, \log S_i^{(6)}) - \text{cov}(\log S_i^{(3)}, \log S_i^{(2)}) \]

\[ \Rightarrow \sigma_3 \sigma_4 \rho_{34} = \sigma_3 \sigma_6 \rho_{36} - \sigma_3 \sigma_2 \rho_{23} \]

\[ = \frac{1}{2} \left( \sigma_3^2 + \sigma_6^2 - \sigma_5^2 \right) - \frac{1}{2} \left( \sigma_3^2 + \sigma_2^2 - \sigma_1^2 \right) \]

\[ = \frac{1}{2} \left( \sigma_1^2 + \sigma_6^2 - \sigma_2^2 - \sigma_5^2 \right) \]

\[ \Rightarrow \rho_{34} = \frac{\sigma_1^2 + \sigma_6^2 - \sigma_2^2 - \sigma_5^2}{2 \sigma_3 \sigma_4} \]

- (1) GBP/USD
- (2) USD/JPY
- (3) GBP/JPY
- (4) EUR/USD
- (5) EUR/GBP
- (6) EUR/JPY
Correlation Risk of Multi-Currency Options

- Correlation coefficients in FX Markets can be inferred from cross instruments
- Correlation risk can be transferred into vega positions
- ... and hence be hedged with vanilla options

\[
\rho_{12} = \frac{\sigma_3^2 - \sigma_1^2 - \sigma_2^2}{2\sigma_1\sigma_2}
\]

\[
\rho_{34} = \frac{\sigma_1^2 + \sigma_6^2 - \sigma_2^2 - \sigma_5^2}{2\sigma_3\sigma_4}
\]
Hedging Correlation Risk with Vanilla Options

Value of a rainbow option: $R(\sigma, \rho)$

Now we know:

Write the value as:

Plain Vega:

Adjusted Vega:

$$\frac{\partial R}{\partial \sigma_i}$$

$$\frac{\partial H}{\partial \sigma_i} = \frac{\partial R}{\partial \sigma_i} + \sum_{j=1}^{5} \sum_{k=j+1}^{6} \frac{\partial R}{\partial \rho_{jk}} \frac{\partial \rho_{jk}}{\partial \sigma_i}$$

$$\frac{\partial \rho_{34}}{\partial \sigma_1} = \frac{\sigma_1}{\sigma_3 \sigma_4}$$

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Extensions

What is the correlation between USD/JPY and GBP/EUR? Done! 😊

Does the law of cosine work in higher dimensions like in a tetrahedron? No doubt! 😊

What else do we need? Nothing 😊

Does the method extend to equity options? Rather not 😞 Use correlation swaps for hedging
How to get the Smile into the Basket?

- Weighted Monte Carlo
- Works well for baskets up to 10 constituents
- Alternative: Optimal Strike Decomposition