Calibrating Spread Options using a Seasonal Forward Model

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1. Outline the pricing of spread options
2. Review a two-factor seasonal commodities model
3. Describe a calibration algorithm based on principal components
4. Present a numerical example of a heating rate option

Background

- Recent surge of interest in commodity derivatives
- In many cases, only the forward prices of the commodity assets are market observables
- Manage exposure to loss through holding commodity derivatives
- Credit risk models must accurately predict the underlying correlated dynamics of the term structure
- Interest rate derivative modeling techniques are useful but limited, e.g. seasonality

Overview of Approaches

1. Ribeiro and Hodges [1] and Barlow et al. [2] respectively apply Kalman filters to calibrate commodity spot prices.
2. Cortazar and Schwartz [3] perform a least squares regression on $lnF_t(T)$.
3. Borovkova and Geman [4] propose a seasonality model for a wider class of seasonal commodity futures.
4. Borovkova and Geman [5] apply principal component analysis to deseasonalized futures prices under the real-world measure.

[1] Diana Ribeiro and Stewart Hodges [2004], A Two-Factor Model for Commodity Prices and Futures Valuation, Technical report, Financial Options Research Center, Warwick Business School.

[2] M. Barlow, Y. Gusev and M. Lai [2004], Calibration of Multifactor Models in Electricity Markets, Int. J. Theo. Appl. Finance, 7(2), pp. 101-120.

[3] G. Cortazar and E.S. Schwartz [2003], Implementing a stochastic model for oil futures, Energy Economics 25, pp. 215â238.

[4] S. Borovkova and H. Geman [2006], Seasonal and stochastic effects in commodity forward curves, Rev Deriv Res (9), pp. 167-186.

[5] S. Borovkova and H. Geman [2006], Analysis and Modelling of Electricity Futures, Studies in Nonlinear Dynamics Econometrics, Volume 10, Issue 3, Article 6.

Risk Neutral Pricing

- The risk neutral price $V(t)$ of a time T expiring option on the spread of two forward contracts is

$$V(t) = exp\{-r\tau\}\mathbb{E}_t^*(|F_1(T,T_1) - F_2(T,T_2) - K|^+)$$

- The risk neutral conditional expectation $\mathbb{E}_t^*(\cdot) := \mathbb{E}^*(\cdot|\mathfrak{F}_t)$
- $F_i(t,T)$ denotes the time t value of a forward contract to deliver the underlying $S(T)$ at time T
- $F_1(t,T_1)$ and $F_2(t,T_2)$ may reference difference underlyings $S_1(t)$ and $S_2(t)$

Examples of Spread Options (I)

Heating rate Option

- The risk neutral price of a heating rate/spark-spread (call) option [a] is

$$V(t) = exp\{-r\tau\}\mathbb{E}_t^*(|F_p(T,T_p) - H_{\text{eff}}F_g(T,T_g) - K|^+)$$

- $F_p(t,T_p)$ is the time $T_p \geq T$ expiring forward power contract
- $F_g(t,T_g)$ is the time $T_g \geq T$ expiring forward natural gas contract
- H_{eff} is a fixed energy efficiency factor
- K is the strike of the option expiring at time T

[a] Daily strip of heating rate options:
$V_t = exp\{-r\tau\}\sum_{m=1,d=1}^{N_m,N_d^m} h_d^m \mathbb{E}_t^*(|F_p(T,T_p) - H_{\text{eff}}F_g(T,T_g) - K|^+)$.

Examples of Spread Options (II)

Calender Spread Option

- The risk neutral price of a calender spread (call) option is

$$V(t) = exp\{-r\tau\}\mathbb{E}_t^*(F(T, T_1) - F(T, T_2) - K|^+)$$

- $F(t, T_i)$ is the time $T_i \geq T$ expiring forward contract
- K is the strike of the option expiring at time T

Kirk's Formula

- There exists a closed form expression for V(t)

$$V(t) = exp\{-r\tau\}(F_2(t,T_2)+K)(F(t,T)N(d_+)-N(d_-)),$$

- where [6]

$$F(t,T;T_1,T_2) := \frac{F_1(t,T_1)}{F_2(t,T_2)+K},$$
$$d_+ = \frac{lnF(t,T)}{\sigma\sqrt{\tau}} + \frac{\sigma\sqrt{\tau}}{2},$$
$$d_- = d_1 - \sigma\sqrt{\tau}.$$

- $F(t,T)$ is *assumed* to be a Martingale w.r.t. the risk neutral measure

$$dF(t,T) = \sigma F(t,T)dW_t^*.$$

[6] $\sigma^2 = \sigma_1^2 + \sigma_2^2 \frac{F_2^2(t)}{F_2(t)+K)^2} - 2\rho\sigma_1\sigma_2 \frac{F_2(t)}{F_2(t)+K}.$

The Story so Far

- The closed form expression for pricing spread options on forward contracts assumes that $F(t,T) = \frac{F_1(t,T_1)}{F_2(t,T_2)+K}$ is log normal under the pricing measure
- We have not yet specificed the dynamics of each forward contract $F_i(t,T_i)$

Historical Natural Gas Forward Prices

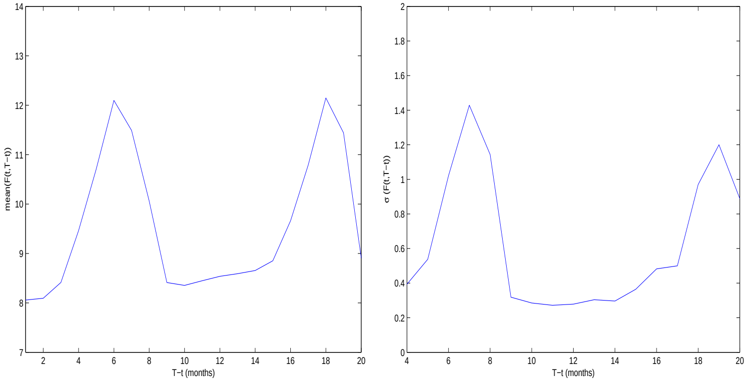


Figure: (Left) Expectation and (right) std. dev. of the Tet M3 natural gas forward curve as a function of monthly maturity date T traded in the month of July.

A Seasonal Forward Model [GEMAN BOROVKOVA]

- Borovkova and Geman [7] express forward prices in component form

$$F_t(T) = \bar{F}_t \exp(s(T) - \gamma_t(\tau)\tau),$$

- $\bar{F}_t$ denotes the mean value of the curve $T \to F(t, T)$ at each time t
- $\gamma_t(\tau)$ is the stochastic convenience yield
- $s(T)$ is the *seasonality* function

[7] S. Borovkova and H. Geman, Seasonal and stochastic effects in commodity forward curves, Rev Deriv Res (9), 2006, pp. 167-186

A Seasonal Forward Model

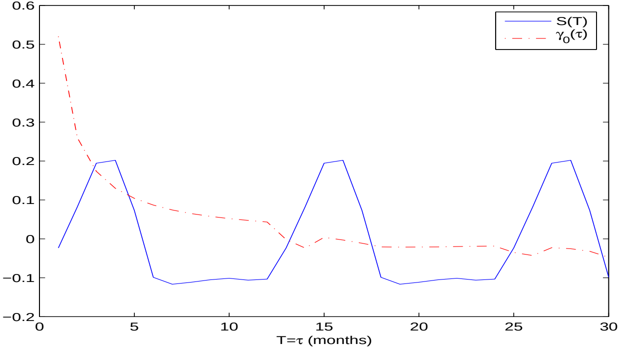


Figure: The historical Tet M3 natural gas forward curve at the start of the time series (where $T = \tau$) is separated into its constitutive components, the seasonality $s(T)$ and the the convenience yield $\gamma_0(\tau)$.

A Seasonal Forward Model

Intrinsic Dynamics

$$d\ln\bar{F}_t = \alpha(m - \ln\bar{F}_t)dt + \sigma dW_t^{[1]}$$
$$d\gamma_t(\tau) = -a(\tau)\gamma_t(\tau) + \eta(\tau)dW_t^{[2]}$$

Two-Factor Forward Model[a]

[a] $a'(\tau) = a(\tau) + 1$

$$d\ln F_t(T) = \left[\alpha(m - \ln\bar{F}_t) + \gamma_t(\tau)a'(\tau)\right]dt + \sigma dW_t^{[1]} - \eta(\tau)\tau dW_t^{[2]},$$

- $W_t^{[1]}$ and $W_t^{[2]}$ are two independent Wiener processes under the *real-world* measure

A Seasonal Forward Model in the Pricing Measure

Risk Neutral Instrinsic Dynamics

$$d\ln \bar{F}_t = \sigma dW_t^{*[1]}$$
$$d\gamma_t(\tau) = \eta(\tau) dW_t^{*[2]}$$

Risk Neutral Two-Factor Forward Model

$$d\ln F_t(T) = \sigma dW_t^{*[1]} - \eta(\tau)\tau dW_t^{*[2]},$$

- $W_t^{*[1]}$ and $W_t^{*[2]}$ are two independent Wiener processes under the *risk neutral* measure

Review of Methodology

1. Compute the geometric average of the futures price [8]

$$ln\bar{F}_t = \frac{1}{N}\sum_{i=1}^{N} lnF_t(T_i)$$

2. Estimate the seasonality function from the historical futures price series

$$\hat{s}(T) = \frac{1}{n}\sum_{i=1}^{n} lnF_{t_i}(T) - ln\bar{F}_{t_i}$$

3. Imply the convenience yield time series from the seasonal forward model

$$\gamma_t(\tau) = \frac{ln\frac{\bar{F}_t}{F_t(T)} - \hat{s}(T)}{\tau}.$$

[8] N is assumed to be a multiple of 12.

The Covariance Matrix

- The theoretical covariance matrix takes the form

$$V_{ij}^{Th} = \int_{t_1}^{t_2} d\gamma_t(T_i) d\gamma_t(T_j)$$

- The implied (empirical) covariance matrix is

$$V_{ij}^{Imp} = \sum_{t=t_1}^{t=t_2-1} \Delta\gamma_t(T_i)\Delta\gamma_t(T_j)$$

Calibration of the Forward Model

Definition (Filtered box constrained calibration problem)

$$\min_{\eta \in S \subset \mathbb{R}_+^N} \hat{z} = |R^T V^{Th} R - \Lambda|_2^2 = \sum_{k,l}^{d \leq N} \left(R_{ki} V_{ij}^{Th} R_{jl} - \delta_{kk} \lambda^k \right)^2$$

- The columns of R and principal diagonal elements of Λ are the eigenvectors and eigenvalues of V^{Imp}

Calibration Algorithm

- $\hat{\eta} = R^T\eta$ is the solution vector projected onto the principal component basis
- Express the gradient $\nabla_\eta \hat{z} = R\nabla_{\hat{\eta}}\hat{z}$
- $\hat{z}$ is everywhere differentiable w.r.t. the projected solution vector
- Specify bounds on the solution vector (not on the projected solution vector)
- Use a gradient-based constrained non-linear optimization algorithm (e.g. projected gradient methods with Armijo rule.)

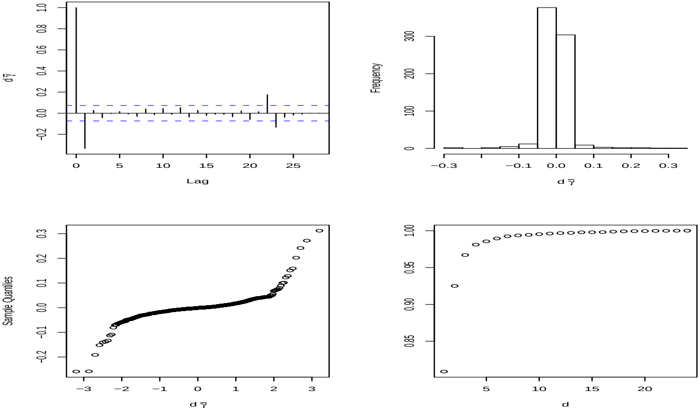
Overview

- Tet M3 and Conn NE natural gas and peak electricity futures prices (USD)
- Montly increments up to two year futures contracts with full historical data over the period Nov-04 to Sep-07
- Perform Shapiro-Wilks and Box-Ljung tests on log returns to measure normality and stationarity
- Compare the performance of numerous constrained optimization algorithms [9] provided in the opensource c++ library Opt++ [10].

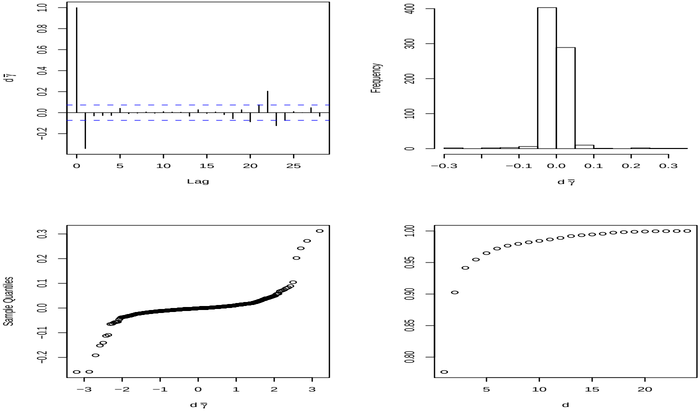
[9] C.T. Kelley [1999], Iterative Methods for Optimization, Frontiers in Applied Mathematics 18, SIAM.

[10] http://csmr.ca.sandia.gov/opt++

Time Series Analysis: Natural Gas Futures



Time Series Analysis: Electricity Futures



Estimated Seasonality

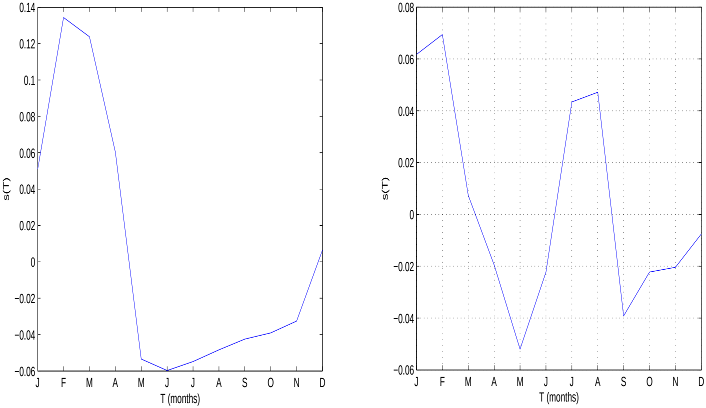
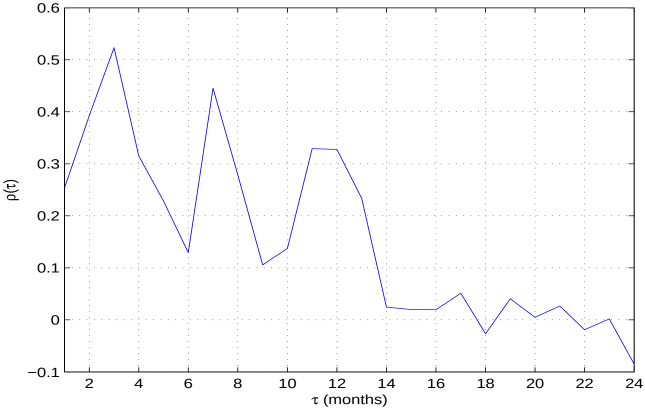


Figure: The seasonality of Tet M3 natural gas and Conn NE peak electricity forwards.

Correlation between Natural gas and Electricity Convenience Yields



Convenience Yield Volatility Term-Structure

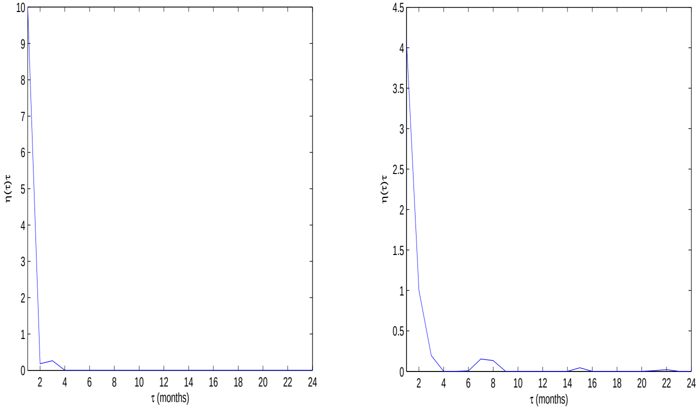


Figure: The calibrated convenience yield volatility term-structure of Tet M3 natural gas and Conn NE peak electricity forwards.

Performance Comparison of Constrained Optimization Algorithms

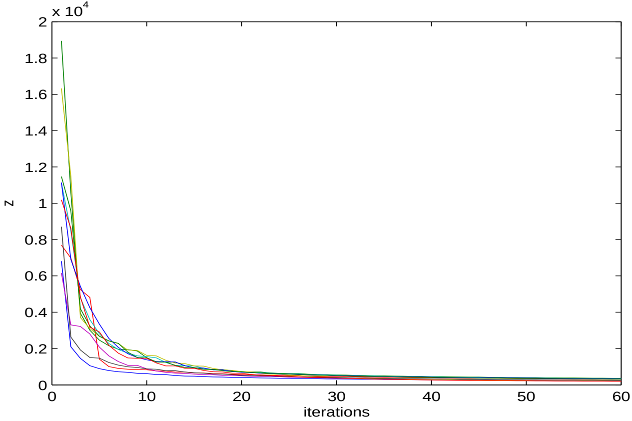


Figure: The gradient projection method.

Performance Comparison of Constrained Optimization Algorithms

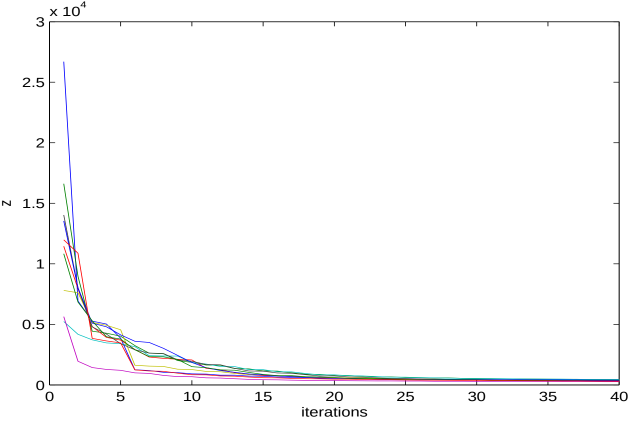


Figure: The projected BFGS method.

Performance Comparison of Constrained Optimization Algorithms

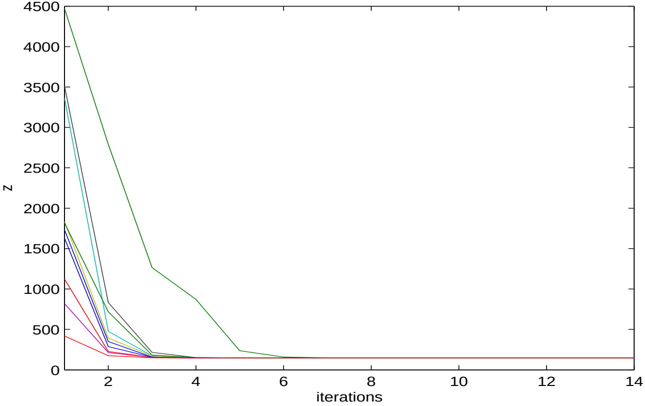


Figure: The interior reflective Newton method.

Performance Comparison of Constrained Optimization Algorithms

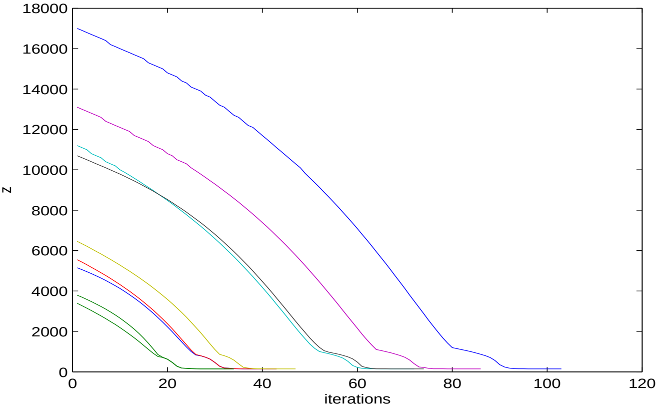


Figure: The finite difference interior point method.

Performance Comparison of Constrained Optimization Algorithms

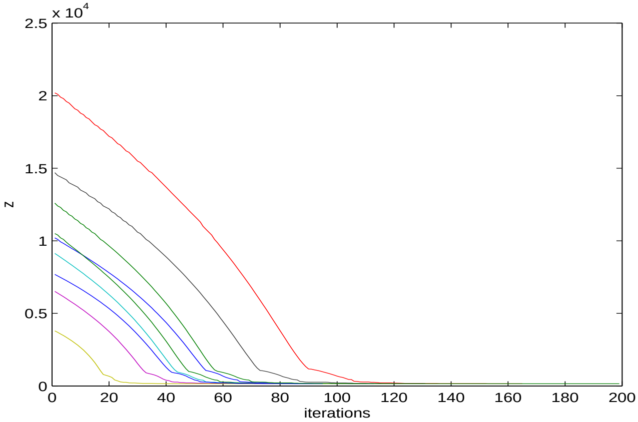


Figure: The quasi-newton interior point method.

Performance Comparison of Constrained Optimization Algorithms

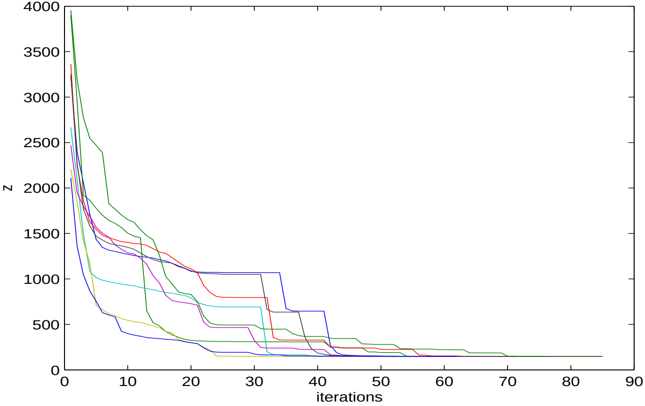
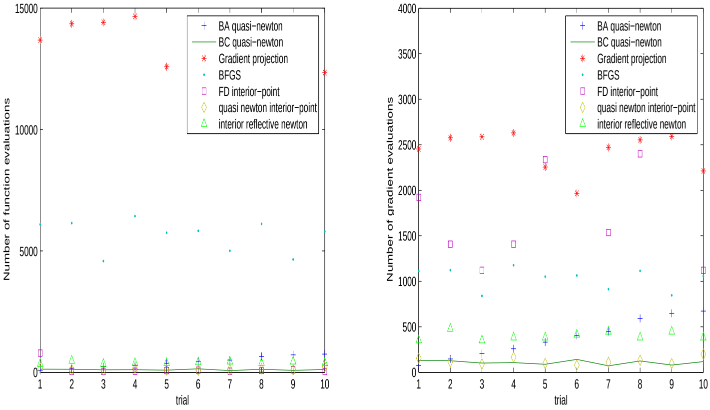
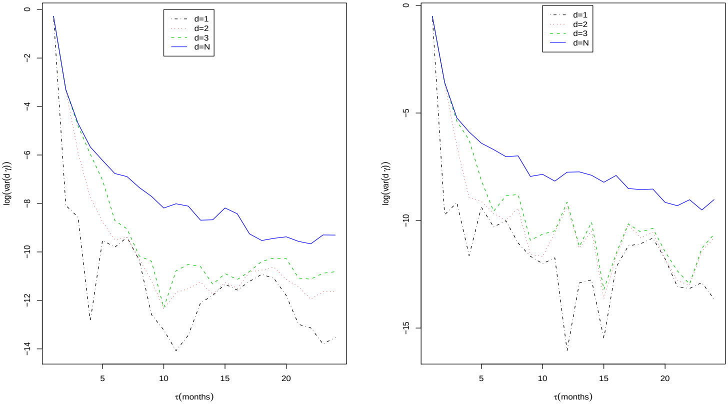


Figure: The BC quasi-newton method.

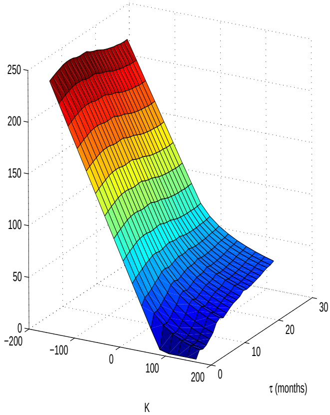
Performance Comparison of Constrained Optimization Algorithms



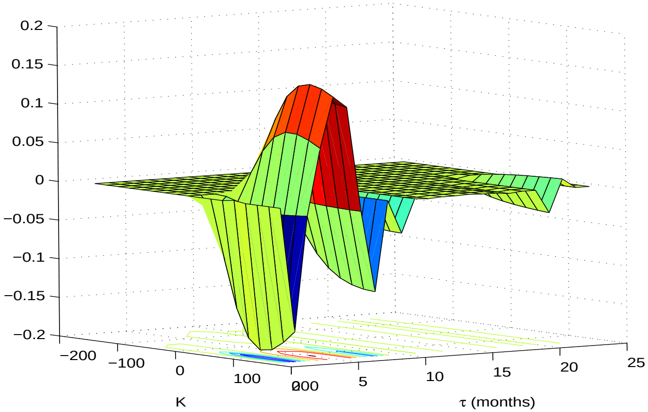
Principal Component Analysis: Natural Gas and Electricity Futures



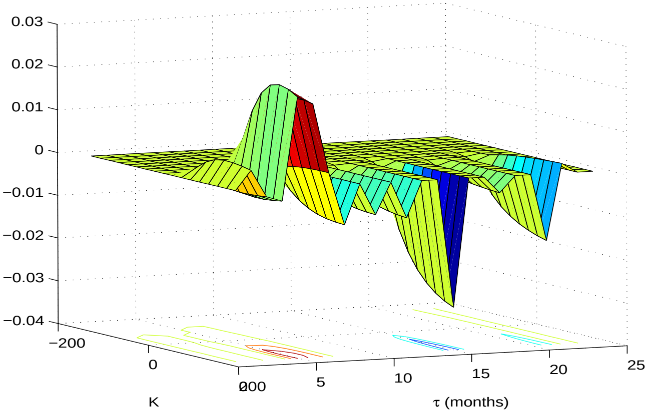
Heating Rate Call Price (USD)



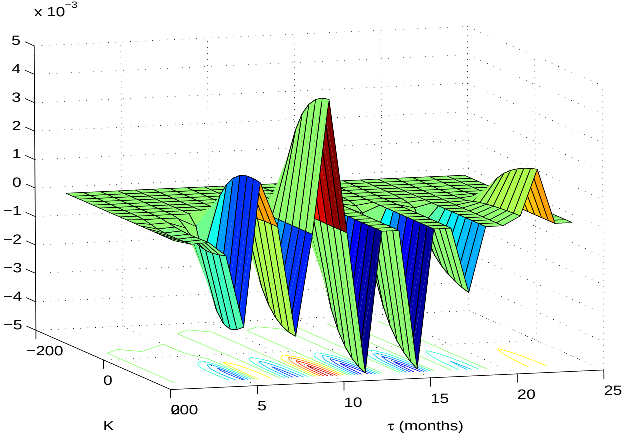
Error in Heating Rate Call Price (USD) with 3 PCs



Error in Heating Rate Call Price (USD) with 5 PCs



Error in Heating Rate Call Price (USD) with 10 PCs



Summary

- The accurate calibration of non-storable spread options to the observed underlying forward contracts is challenging
- First deseasonalize historical time series of log returns and perform PCA on the correlated convenience yield returns
- The volatility term-structure can be captured with only a few principal components
- Preliminary results suggest that the combination of a seasonal forward model, PCA and a gradient based constrained optimization algorithm is efficient and robust (avoid simulated annealing/genetic algorithms)
- Future directions:
    - Automate the selection of the number of principal components according to errors in the greeks.
    - Fit uncorrelated GARCH processes for the volatility w.r.t. each of the principal components.