

# Optimization with PDEs in the presence of constraints - tailored discrete concepts and error analysis

**K. Deckelnick<sup>1</sup>, A. Günther<sup>2</sup>, and M. Hinze<sup>2</sup>,**

<sup>1</sup> Fakultät für Mathematik, Universität Magdeburg

<sup>2</sup> Department Mathematik, Universität Hamburg

e-mail: Klaus.Deckelnick@Mathematik.Uni-Magdeburg.DE,  
Andreas.Guenther@math.uni-hamburg.de, Michael.Hinze@uni-hamburg.de

## ABSTRACT

We discuss tailored discretization concepts for optimal control problems with elliptic and parabolic pdes in the presence of constraints. In particular we consider

- control constraints combined with state constraints [3,4], and
- control constraints combined with constraints on the gradient of the state [1].

We consider distributed control as well as Neumann and Dirichlet boundary control [5] and prove optimal error bounds on the primal optimization variables. Furthermore, we present an extension of the DWR concept to control and/or state constrained optimal control problems which among other things avoids the numerical evaluation of differences formed by multipliers [2]. Finally we present numerical examples which confirm our analytical findings.

## REFERENCES

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