# Characterization of CLTs for Ergodic Markov Chains via Regeneration

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***Abstract.*** Let $(X_n)_{n\geqslant 0}$ be a Markov chain on $(\mathcal{X}, \mathcal{B}(\mathcal{X}))$ with transition kernel $P$ and stationary distribution $\pi$. We say that $(X_n)_{n\geqslant 0}$ is ergodic if

$$\lim_{n\to\infty} \|P^n(x,\cdot) - \pi\|_{tv} = 0, \quad \text{for every} \quad x \in \mathcal{X},$$

where $\|\cdot\|_{tv}$ is the total variation distance.

For a function $g : \mathcal{X} \to \mathbb{R}$ with $\pi g^2 < \infty$, define $S_n = \sum_{i=0}^{n-1} \bar{g}(X_i)$, where $\bar{g} = g - \pi g$. We say that a central limit theorem holds for $(X_n)_{n\geqslant 0}$ and $g$ if

$$\frac{S_n}{\sqrt{n}} \xrightarrow{d} N(0, \sigma_g^2), \quad \text{for some} \quad \sigma_g^2 < \infty. \tag{1}$$

CLTs for ergodic Markov chains are of crucial practical importance in sensible implementing of MCMC algorithms and of vital theoretical interest.

However, ergodic chains admit the split chain construction (cf. [2]) with an atom $\check{\alpha} \in \mathcal{X} \times \{0,1\}$ of positive stationary measure. Let $\sigma_{\check{\alpha}}(0), \sigma_{\check{\alpha}}(1), \ldots,$ denote consecutive regeneration times (visits of $(X_n)_{n\geqslant 0}$ to $\check{\alpha}$). By

$$Z_1, Z_2, ...$$

denote consecutive tours between regenerations. $Z_1, Z_2, ...$ are identically distributed random variables. We present the following

**Theorem 0.1.** *A CLT holds for $(X_n)_{n\geqslant 0}$ and $g$, if and only if, $EZ_1^2 < \infty$.*

## References

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