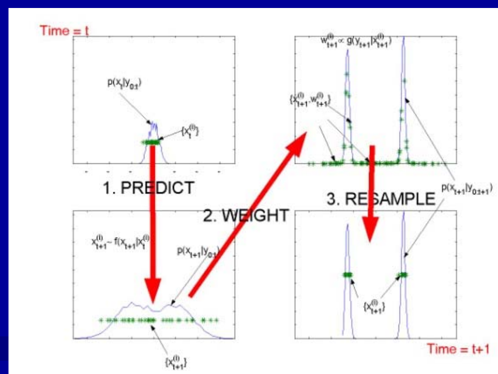
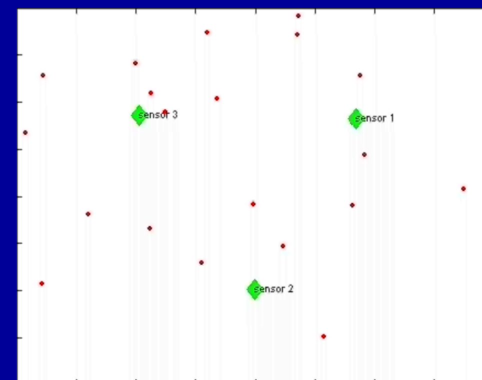
# *Sequential inference for dynamically evolving groups of objects*

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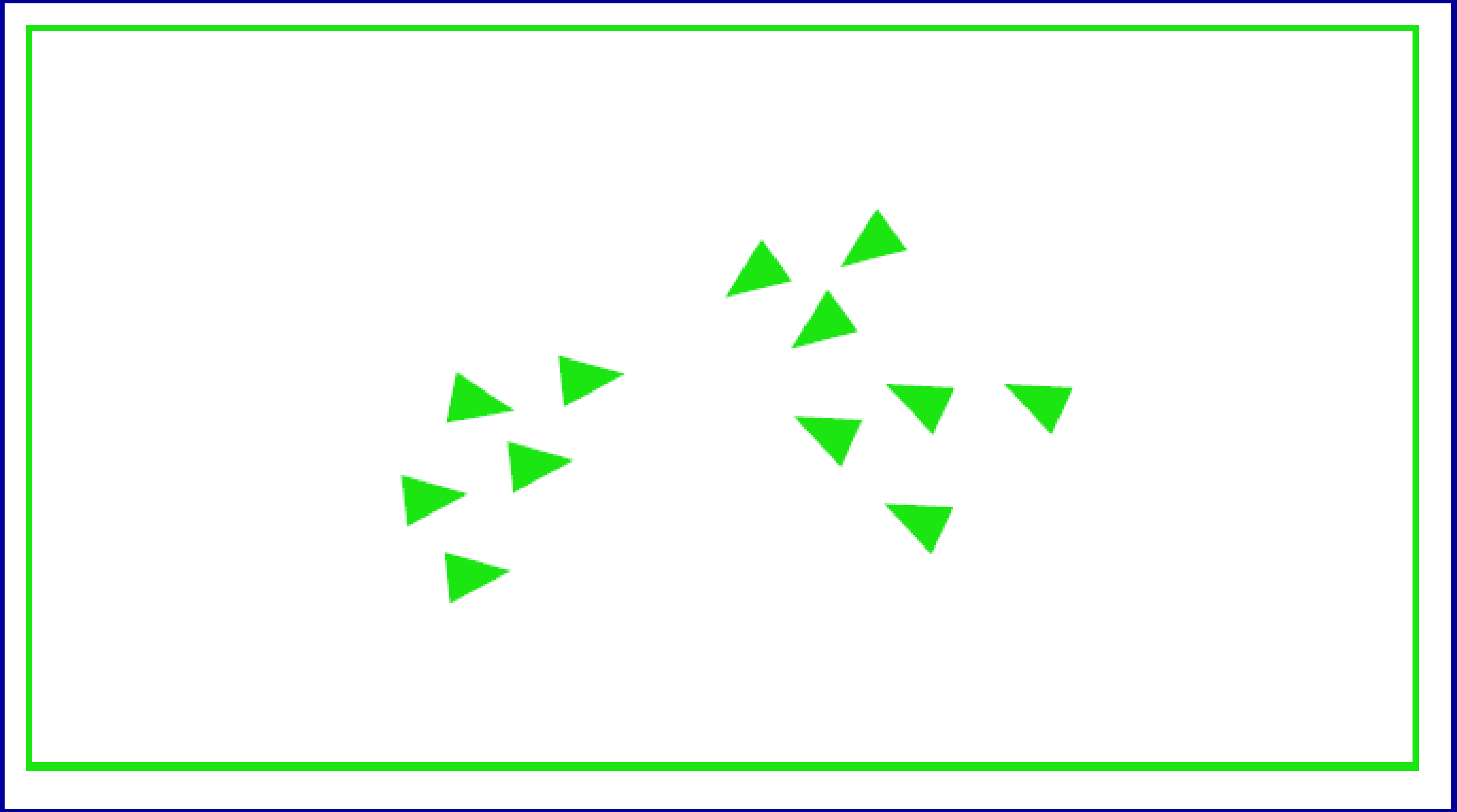
# Overview

- The Group Tracking Problem
- Monte Carlo Filtering for high-dimensional problems
- Stochastic models for groups
- Inference algorithm
- Results
- Future directions

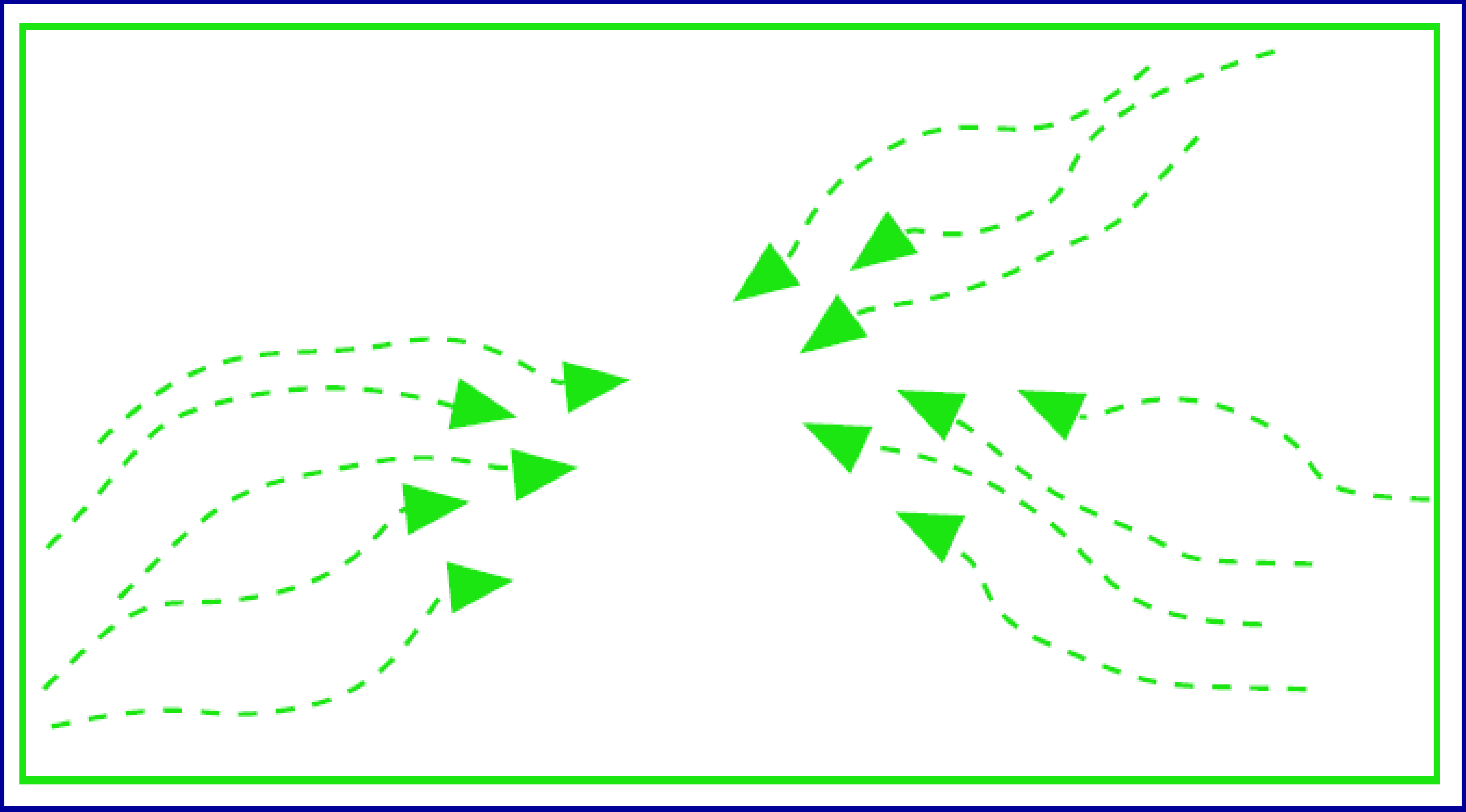
# Group Tracking

- For many surveillance applications, targets of interest tend to travel in a group - groups of aircraft in a tight formation, a convoy of vehicles moving along a road, groups of football fans, dynamic clusterings in finance and medical applications.
- This group information can be used to improve detection and tracking. Can also help to learn higher level behavioural aspects and intentionality.
- Some tracking algorithms do exist for group tracking. However implementation problems resulting from the splitting and merging of groups have hindered progress in this area [see e.g. Blackman and Popoli 99].
- This work develops a group model and algorithms for joint inference of targets' states as well as their group structures – both may be dynamic over time (splitting/merging, breakaway...)

# Standard multi-object tracking problem:



# Dynamic group-based problem:



At time $t$, $X_t$ represents all the individual objects' states. We will use $G_t$ to represent the group structure or configuration. $Z_t$ will represent the observations.

Observations and objects are assumed Markovian as in standard tracking problems.

The joint distribution of the dynamic group tracking model can be written as:

State dynamics

Group dynamics

$$p(X_{0:t}, G_{0:t}, Z_{1:t}) =$$

$$p(X_0|G_0)p(G_0) \prod_{k=1}^{t} p(X_k|X_{k-1}, G_k, G_{k-1})p(G_k|G_{k-1}, X_{k-1})p(Z_k|X_k)$$

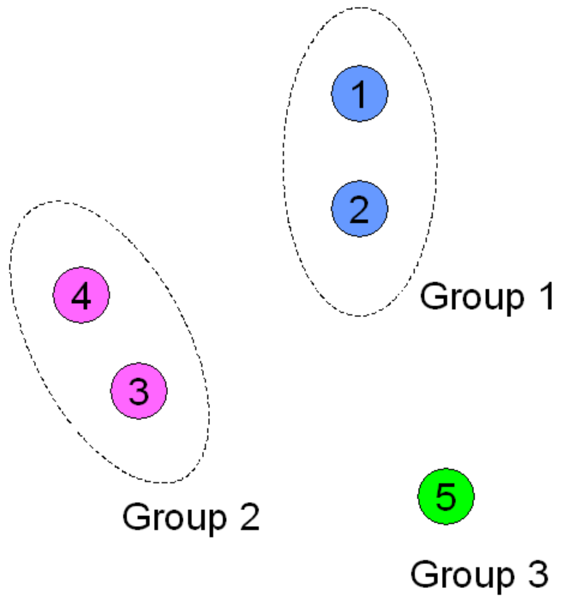
Initial state prior

Likelihood

[We will later augment also with an existence variable $e_k$ for each object]

# Group variable G

$$G_t = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \\ 3 \end{bmatrix}$$



# Inference objective

Wish to infer the joint posterior distribution of $G_t$ and $X_t$.

$$p(X_{0:t}, G_{0:t} | Z_{1:t}) \propto p(X_{0:t}, G_{0:t}, Z_{1:t})$$

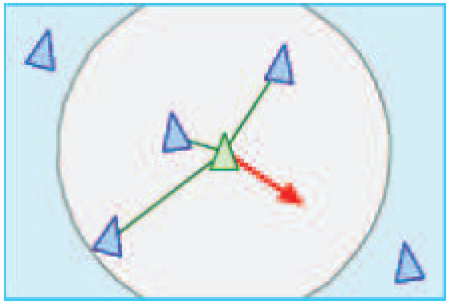
This will be carried out sequentially using a sequential MCMC algorithm (cf particle filters)

# Stochastic models for groups

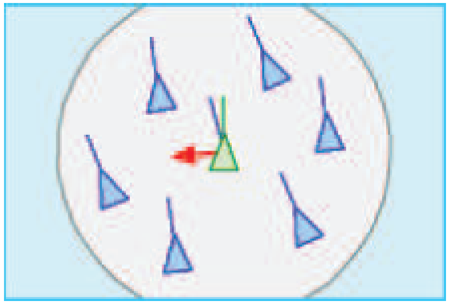
$$p(X_k | X_{k-1}, G_k, G_{k-1})$$

- Require dynamical models that adequately capture the correlated behaviour of group objects
- We base this on simple behavioural properties of individuals relative to other members of their group (attractive/repulsive forces)
- Some similarities to flocking models in animal behaviour analysis

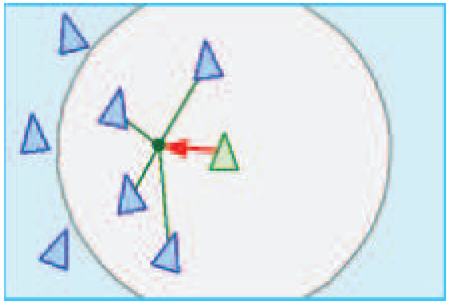
# Science of Flocking



Separation:
steer to avoid
crowding local
flockmates



Alignment:
steer towards the
average heading of
local flockmates



Cohesion:
steer to move
toward the average
position of local
flockmates

Craig Reynolds - `http://www.red3d.com/cwr/boids/`

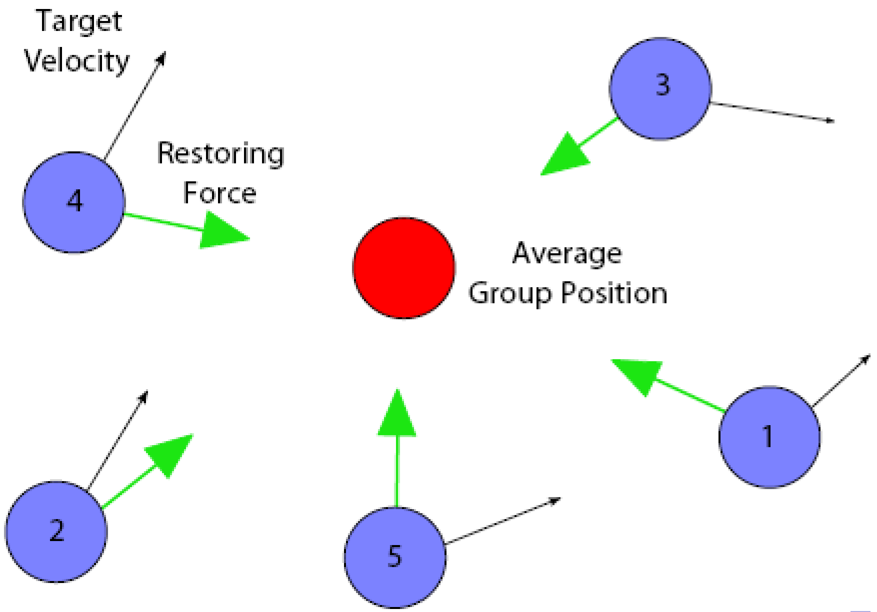
# Continuous Time Group Model

- We can write down a specially constructed SDE:

$$
\begin{aligned}
\mathrm{d}\dot{s}_{t,i} \quad &= \quad \left\{ -\alpha\Big[s_{t,i} - \frac{1}{N_{\Upsilon_u}} \sum_{j \in \Upsilon_u} s_{t,j}\Big] - \gamma \dot{s}_{t,i} - \beta\Big[\dot{s}_{t,i} - \frac{1}{N_{\Upsilon_u}} \sum_{j \in \Upsilon_u} \dot{s}_{t,j}\Big] \right\} \mathrm{d}t \\
&\quad + \mathrm{d}W^s_{t,i} + \mathrm{d}B^s_t \qquad (2)
\end{aligned}
$$

# Continuous Time Group Model

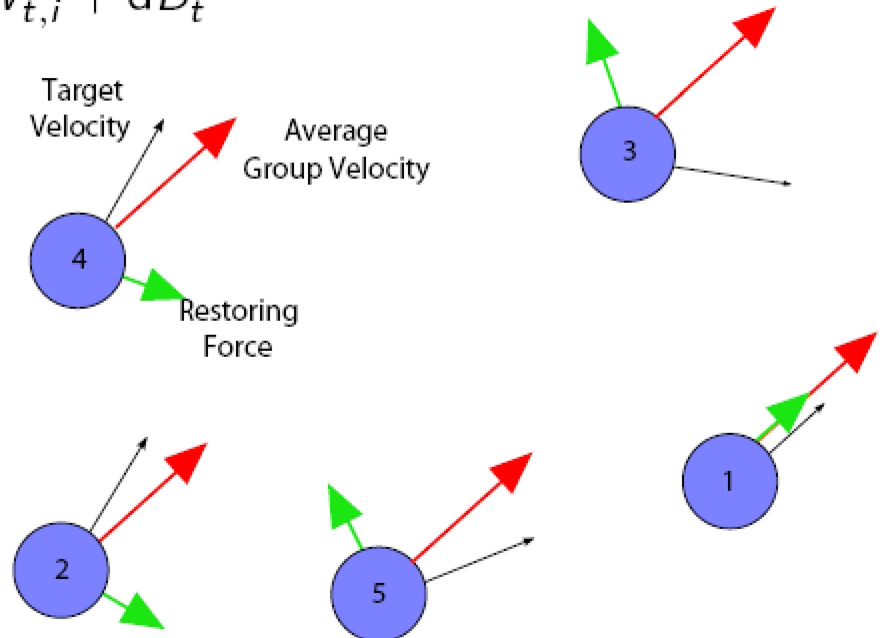
$$d\dot{s}_{t,i} = \left\{ \underbrace{-\alpha\Big[s_{t,i} - \frac{1}{N_{\Upsilon_u}} \sum_{j \in \Upsilon_u} s_{t,j}\Big]}^{\text{Restoring force towards average position}} - \gamma \dot{s}_{t,i} - \beta\Big[\dot{s}_{t,i} - \frac{1}{N_{\Upsilon_u}} \sum_{j \in \Upsilon_u} \dot{s}_{t,j}\Big] \right\} dt$$
$$+ dW^{s}_{t,i} + dB^{s}_{t}$$



# Continuous Time Group Model

$$
d\dot{s}_{t,i} = \left\{ -\alpha\left[s_{t,i} - \frac{1}{N_{\Upsilon_u}} \sum_{j\in\Upsilon_u} s_{t,j}\right] - \gamma \dot{s}_{t,i} \overbrace{- \beta\left[\dot{s}_{t,i} - \frac{1}{N_{\Upsilon_u}} \sum_{j\in\Upsilon_u} \dot{s}_{t,j}\right]}^{\text{Restoring force towards average velocity}} \right\} dt
$$

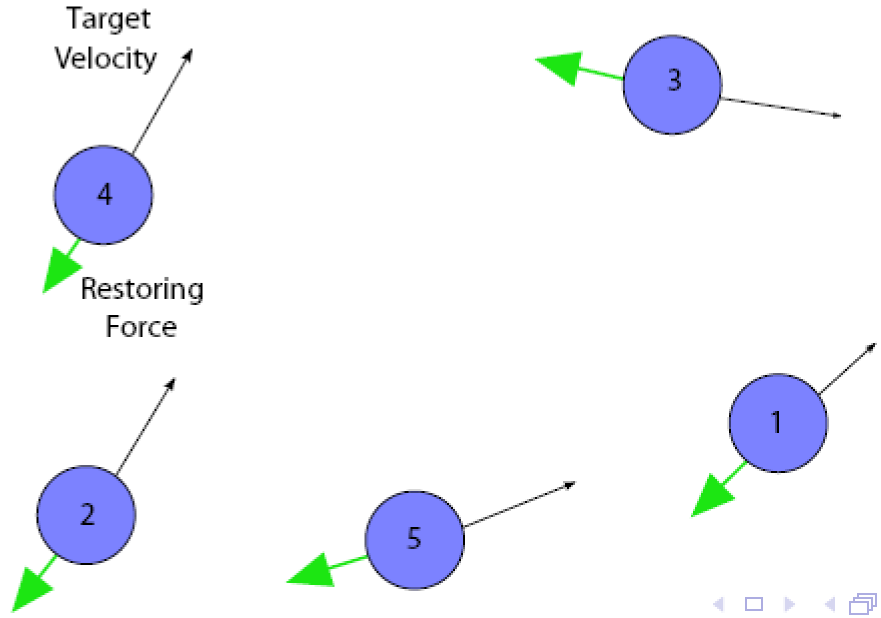
$$
+ dW^{s}_{t,i} + dB^{s}_{t}
$$



# Continuous Time Group Model

$$
d\dot{s}_{t,i} = \left\{ -\alpha\Big[s_{t,i} - \frac{1}{N_{\Upsilon_u}} \sum_{j\in\Upsilon_u} s_{t,j}\Big] - \overbrace{\gamma \dot{s}_{t,i}}^{\text{Prevents excessive speed}} - \beta\Big[\dot{s}_{t,i} - \frac{1}{N_{\Upsilon_u}} \sum_{j\in\Upsilon_u} \dot{s}_{t,j}\Big] \right\} dt
$$

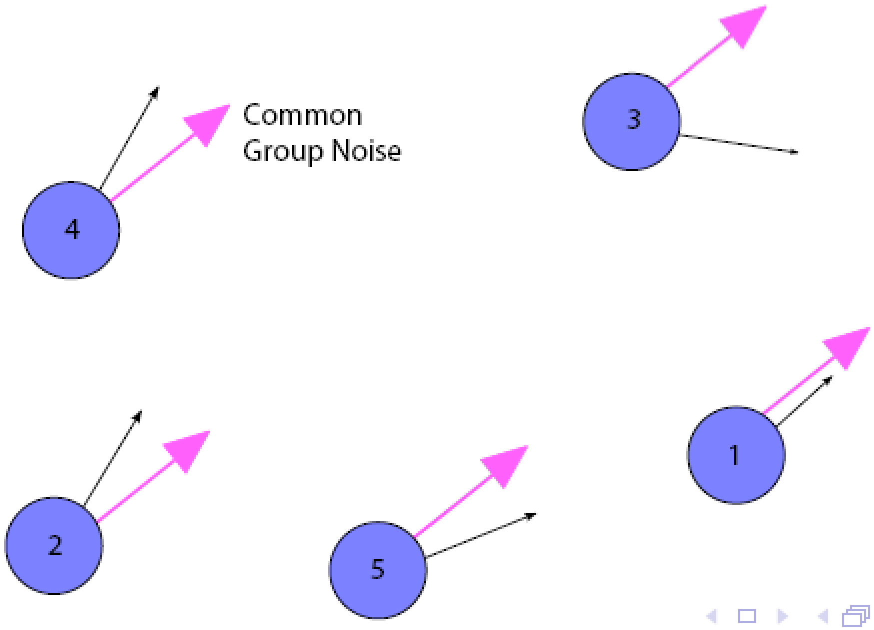
$$
+ dW^s_{t,i} + dB^s_t
$$



# Continuous Time Group Model

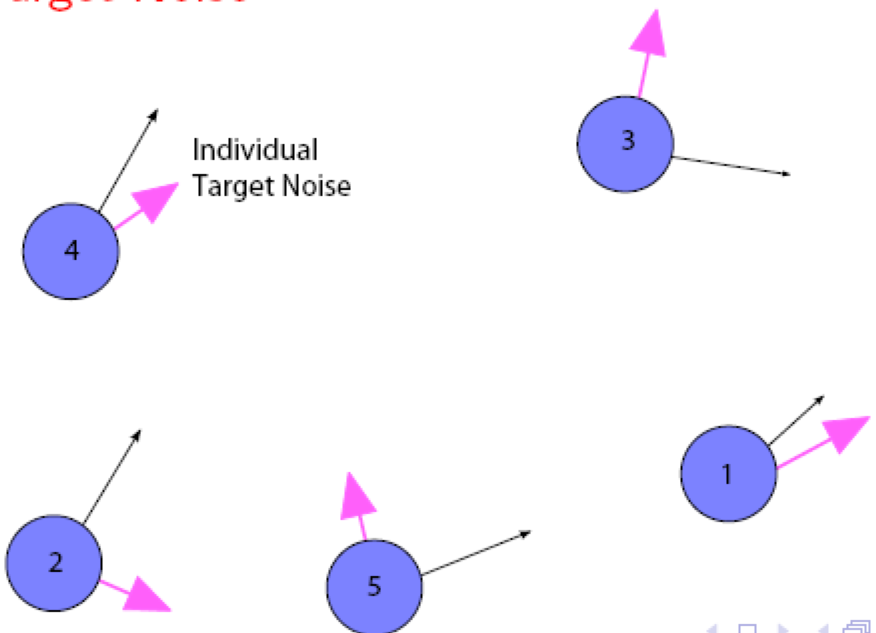
$$d\dot{s}_{t,i} = \left\{ -\alpha[s_{t,i} - \frac{1}{N_{\Upsilon_u}} \sum_{j \in \Upsilon_u} s_{t,j}] - \gamma \dot{s}_{t,i} - \beta[\dot{s}_{t,i} - \frac{1}{N_{\Upsilon_u}} \sum_{j \in \Upsilon_u} \dot{s}_{t,j}] \right\} dt$$

$$+dW^s_{t,i} + \underbrace{dB^s_t}_{\text{Common Group Noise}}$$



# Continuous Time Group Model

$$
\begin{aligned}
\mathrm{d}\dot{s}_{t,i} &= \left\{ -\alpha[s_{t,i} - \frac{1}{N_{\Upsilon_u}} \sum_{j \in \Upsilon_u} s_{t,j}] - \gamma \dot{s}_{t,i} - \beta[\dot{s}_{t,i} - \frac{1}{N_{\Upsilon_u}} \sum_{j \in \Upsilon_u} \dot{s}_{t,j}] \right\} \mathrm{d}t \\
&\quad \underbrace{+\mathrm{d}W^s_{t,i}}_{\text{Individual Target Noise}} + \mathrm{d}B^s_t
\end{aligned}
$$



## Continuous Time Group Model

- The SDE can be solved exactly to obtain a linear and Gaussian model of the form

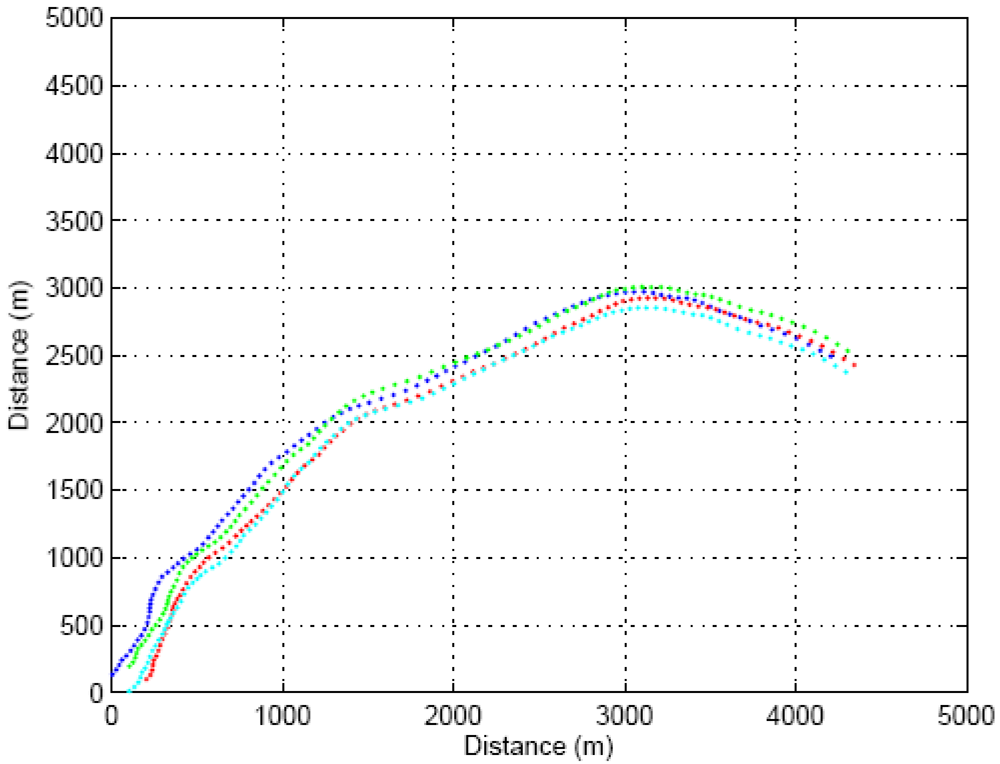
$$p_u(X_{t,\Upsilon_u} | X_{t-1,\Upsilon_u}) = \mathbf{N}(F_{N_{\Upsilon_u}} X_{t-1,\Upsilon_u}, \Sigma_{u,N_{\Upsilon_u}}) \tag{3}$$

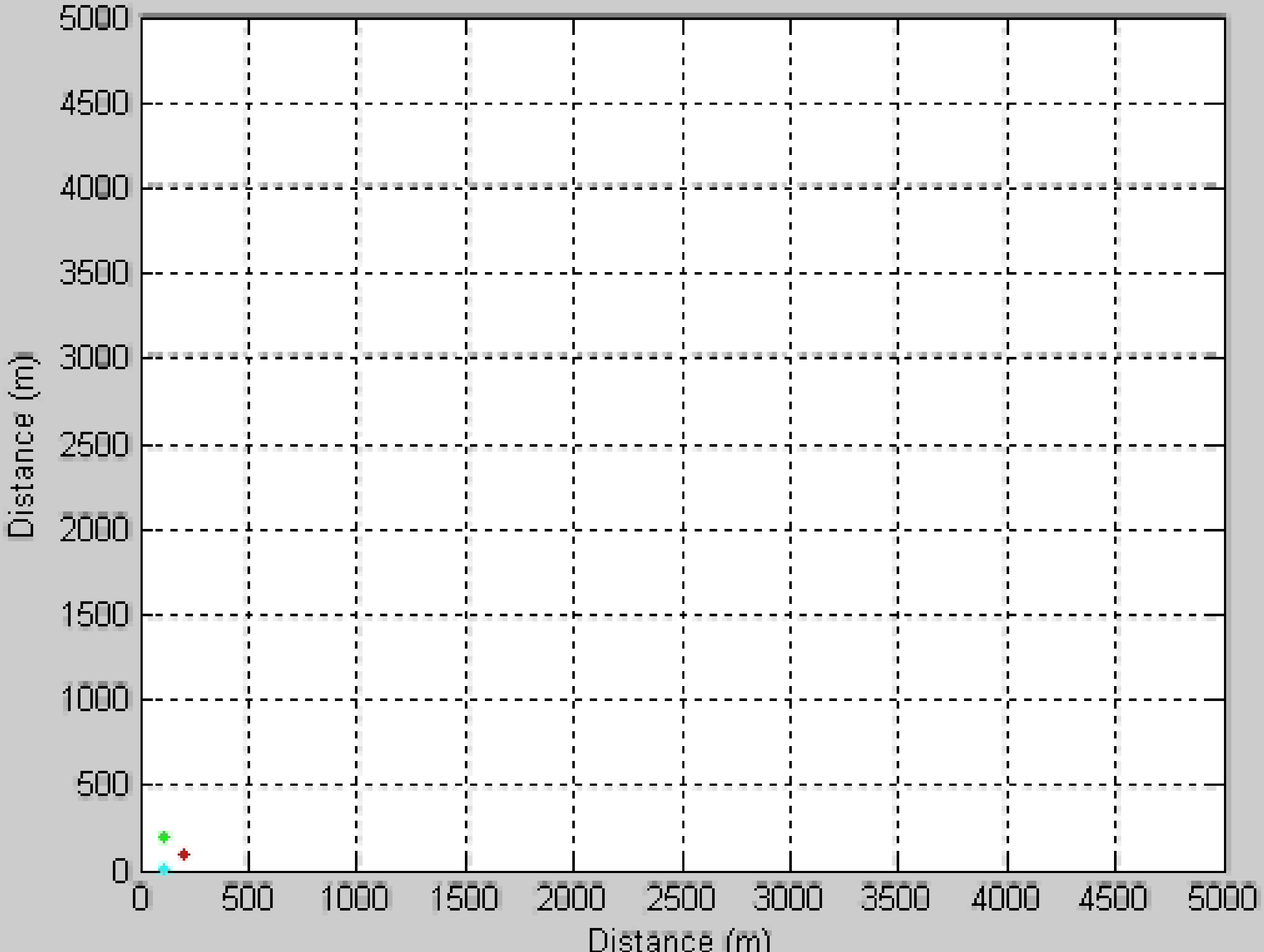
where $F_{N_{\Upsilon_u}} = exp(A\tau)$ and $\Sigma_{u,N_{\Upsilon_u}}$ can be solved using Matrix Fraction Decomposition.

Also include a repulsion mechanism for close targets which discourages collisions.

# Example Group Tracks

- One group of four targets





5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0
Distance (m)
0 500 1000 1500 2000 2500 3000 3500 4000 4500 5000
Distance (m)

# State variables

- We will assume that we are required to track up to a maximum of $N_{\max}$ targets.
- For each target $i$, we will represent its state at time $t$ as $X_{t,i} = [x_{t,i}\ \dot{x}_{t,i}\ y_{t,i}\ \dot{y}_{t,i}]^T$ and use an existence variable $e_{t,i} \in \{0, 1\}$ to represent whether the target is active or not. $e_t$ is modelled as a discrete Markov chain.
- At each time $t$, the group structure is represented by a variable $G_t$.

# Bayesian filtering recursions

Assuming a Markovian state transition, the standard Bayesian filtering prediction and update steps are given by

$$\begin{aligned} & p(X_t, e_t, G_t | Z_{1:t-1}) = \\ & \int p(X_t, e_t, G_t | X_{t-1}, e_{t-1}, G_{t-1}) \times \\ & p(X_{t-1}, e_{t-1}, G_{t-1} | Z_{1:t-1}) dX_{t-1} de_{t-1} dG_{t-1} \end{aligned} \tag{1}$$

$$p(X_t, e_t, G_t | Z_{1:t}) = \frac{p(Z_t | X_t, e_t, G_t) p(X_t, e_t, G_t | Z_{1:t-1})}{p(z_t | Z_{1:t-1})} \tag{2}$$

where $Z_{1:t} = [Z_1 \ \cdots \ Z_m \ \cdots Z_t]$ and $Z_m$ are all the observations collected at time step $m$.

# State Transition Probabilities

- We choose to expand the transition probability model as

$$\begin{aligned} & p(X_t, e_t, G_t | X_{t-1}, e_{t-1}, G_{t-1}) = \\ & p(X_t, e_t | G_t, X_{t-1}, e_{t-1}) p(G_t | X_{t-1}, e_{t-1}, G_{t-1}) \end{aligned} \quad (1)$$

- $p(G_t | X_{t-1}, e_{t-1}, G_{t-1})$ is the discrete jump probability from one group structure to another.
- $p(X_t, e_t | G_t, X_{t-1}, e_{t-1})$ is the target dynamical model given its group structure $G_t$ and it is designed such that it is unaffected by the group structure at time $t-1$.

# Inference Algorithm

- We require a powerful scheme that is sequential and able to sample a high-dimensional, structured state-space
- We adopt a sequential MCMC scheme that samples from the joint states at t and t-1, based on the empirical filtering distribution at time t-1

Target the joint distribution of current ($t$) and previous ($t-1$) states.
Denote $\alpha_t = [X_t,\, e_t,\, G_t]$.
Then sequential target distribution is:

$$p(\alpha_t, \alpha_{t-1}|Z_{1:t}) \propto p(Z_t|\alpha_t)p(\alpha_t|\alpha_{t-1})p(\alpha_{t-1}|Z_{1:t-1})$$

Approximate $p(\alpha_{t-1}|Z_{1:t-1})$ with its empirical distribution from the converged MCMC run at $t-1$:

$$p(\alpha_{t-1}|Z_{1:t-1}) \approx \sum_{i=1}^{N} \delta_{\alpha_{t-1}^{(i)}}(\alpha_{t-1})$$

Then run a new MCMC targetting $p(\alpha_t, \alpha_{t-1}|Z_{1:t})$, etc...

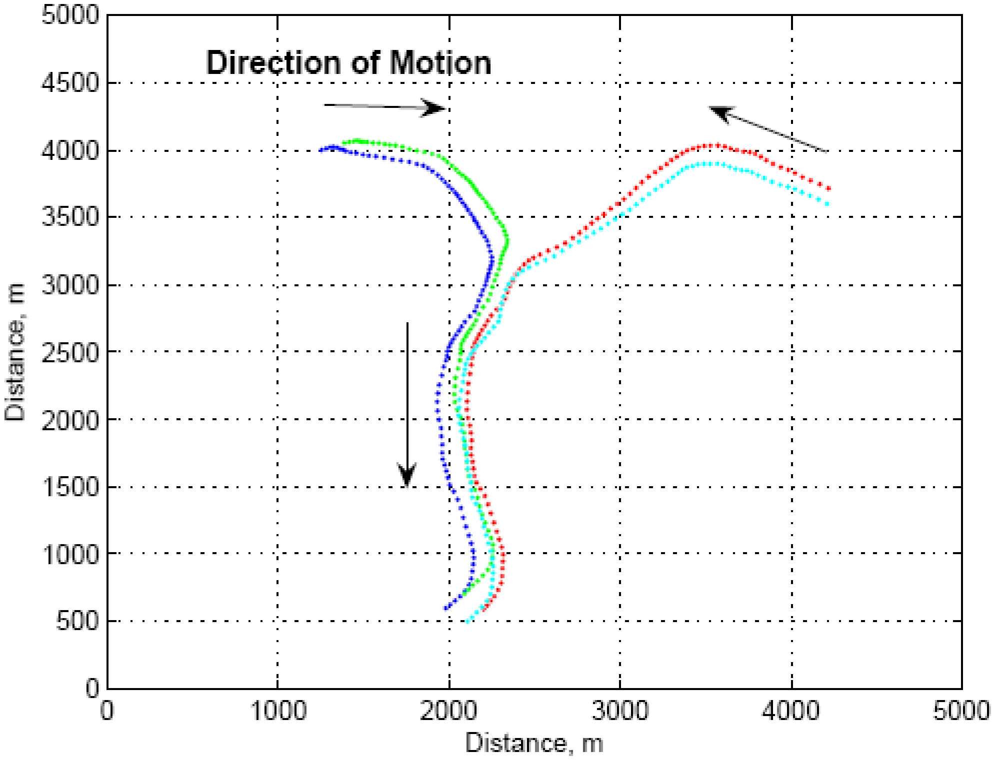
Advantages over importance sampling approaches - no weighting or resampling - can use full power of MCMC sampling $\rightarrow$ high dimensional, structured state spaces.

[See Berzuini et al. 1997, Khan et al. 2006, Golightly and Wilkinson 2006]

There are powerful adaptations to this basic approach - using population/multiple chain MCMC, genetic operations, etc. see Carmi, Septier and Godsill (2009 - submitted).

# Simulation Scenario 1

- Two groups of two targets each merging



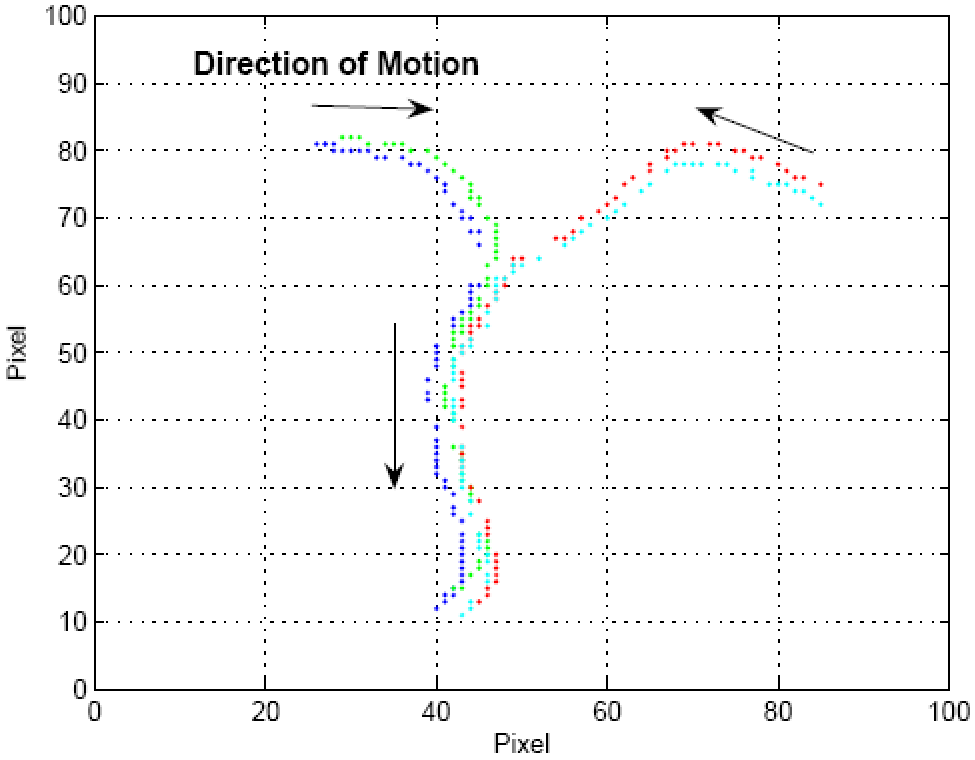
# Simulation Scenario 1

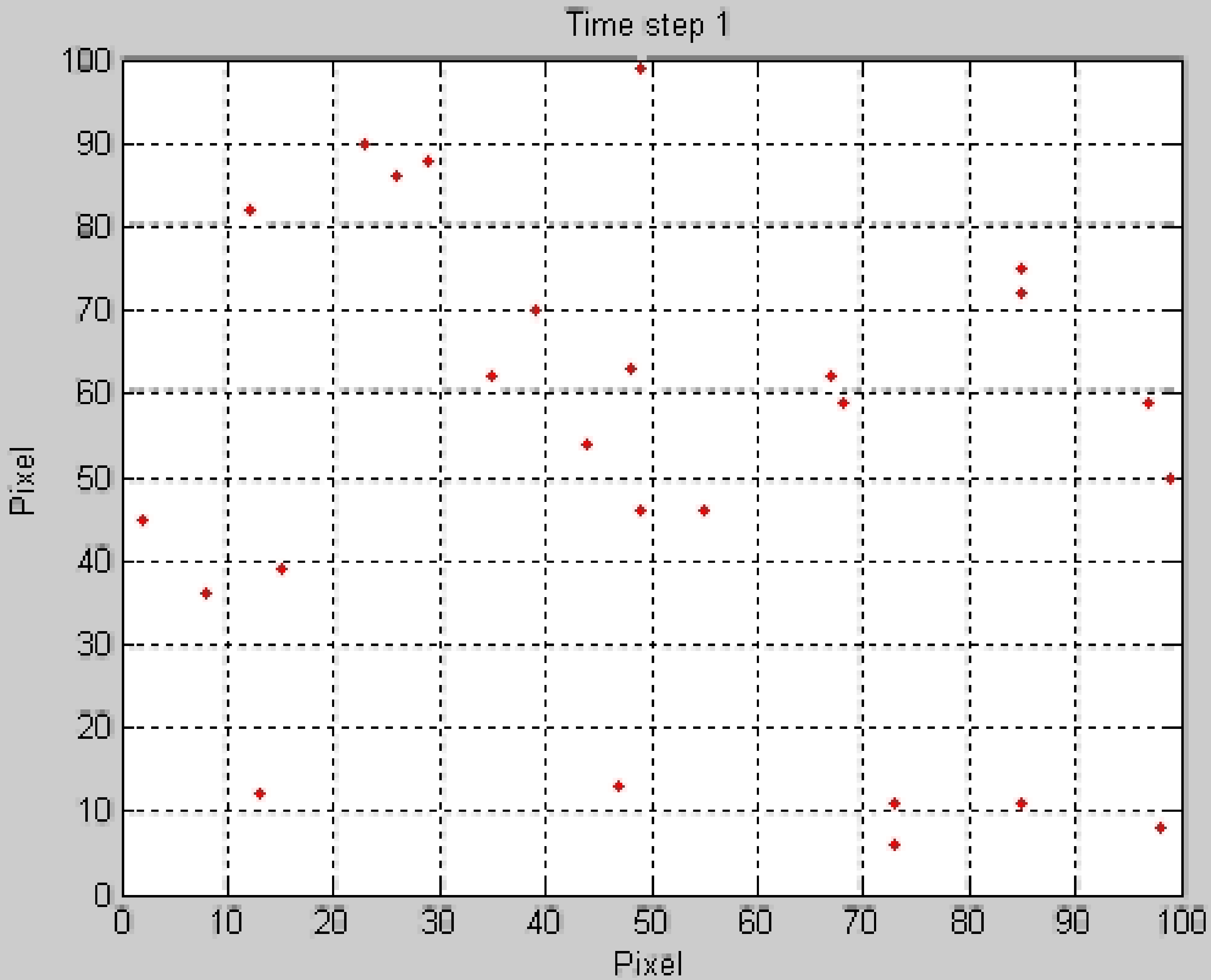
| Simulation Parameter | Symbol | Value |
|---|---|---|
| Probability of detection (for one target) | $P_{d,1}$ | 0.7 |
| Probability of false alarm | $P_{fa}$ | 0.002 |
| Size of scene | $M_x$ by $M_y$ | $100 \times 100$ pixels |
| Size of pixel | $D_x$ by $D_y$ | 50m$\times$50m |
| Time interval between measurements | $\tau$ | 5 seconds |
| Actual number of targets | N | 4 |
| Number of simulation time steps | | 100 |
| Centroid Control Parameter | $\alpha$ | 0.0006 |
| Group Velocity Control Parameter | $\beta$ | 0.3 |
| Individual Velocity Control Parameter | $\gamma$ | 0.001 |
| Individual motion noise | $\sigma_x$,$\sigma_y$ | 0.8 |
| Group motion noise | $\sigma_g$ | 0.5 |

Table: Tracking Parameters

# Simulation Scenario 1

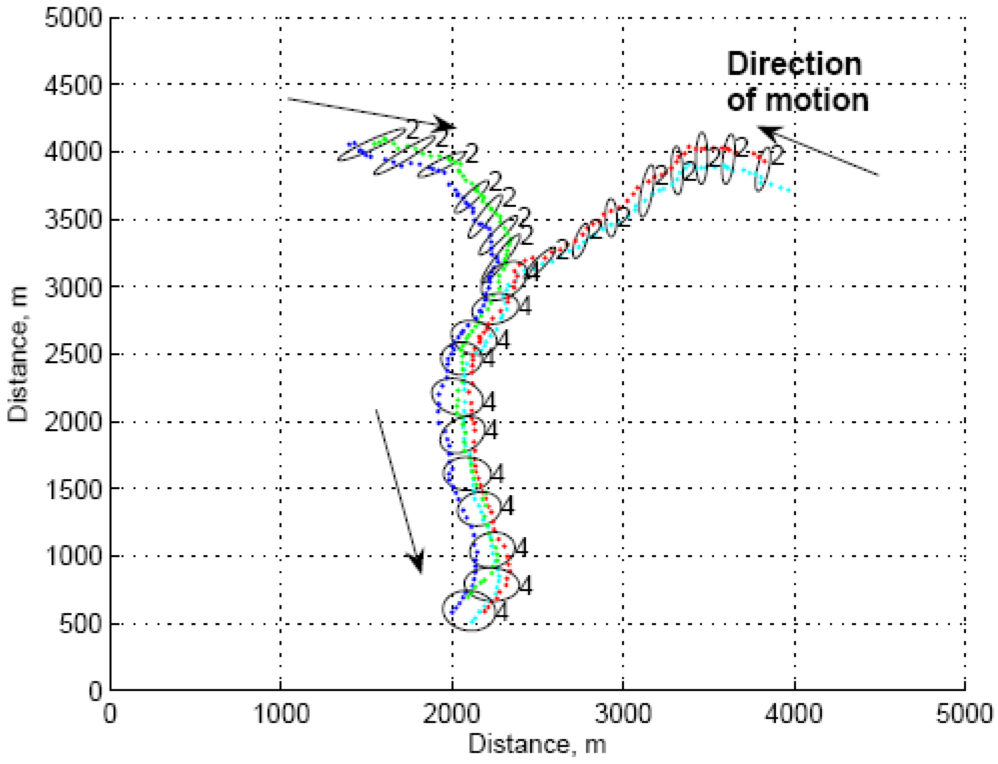
- 30 sets of observations are generated with the same tracks.



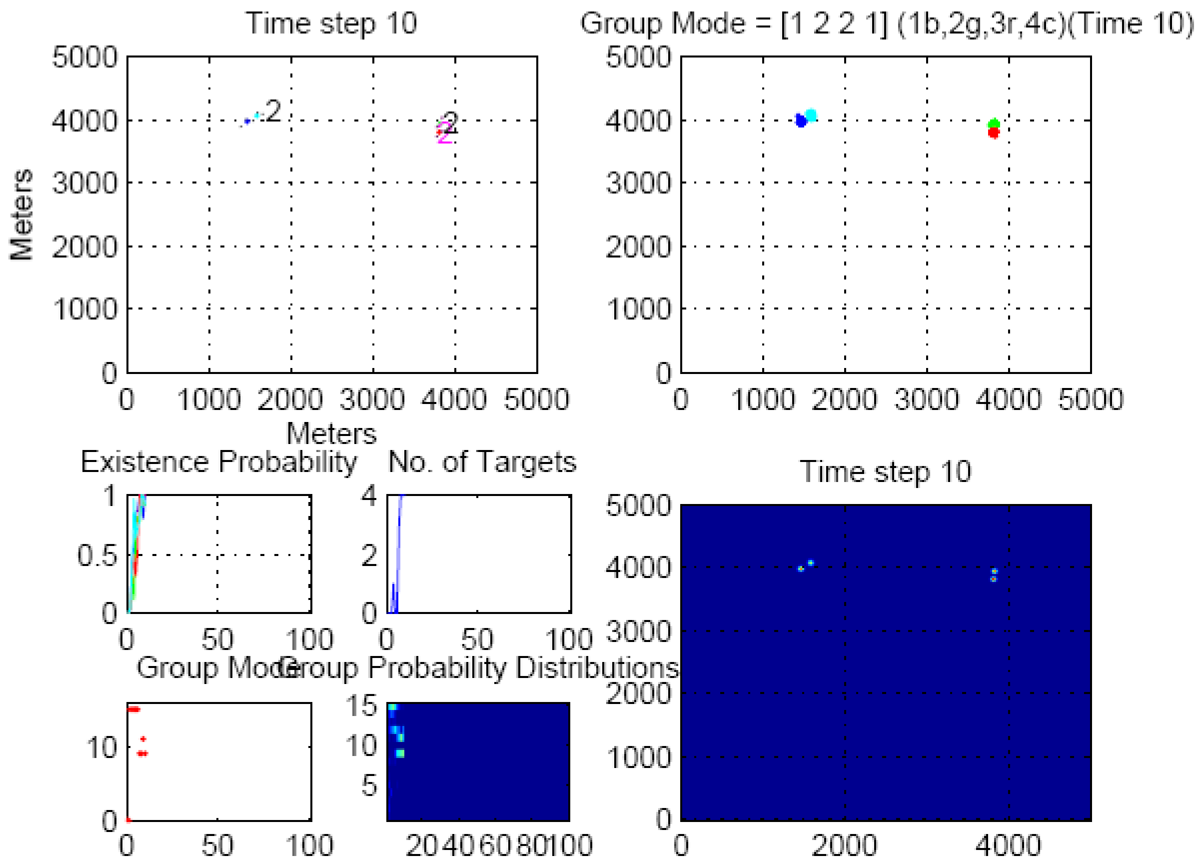

Time step 1
Pixel
Pixel
0
10
20
30
40
50
60
70
80
90
100

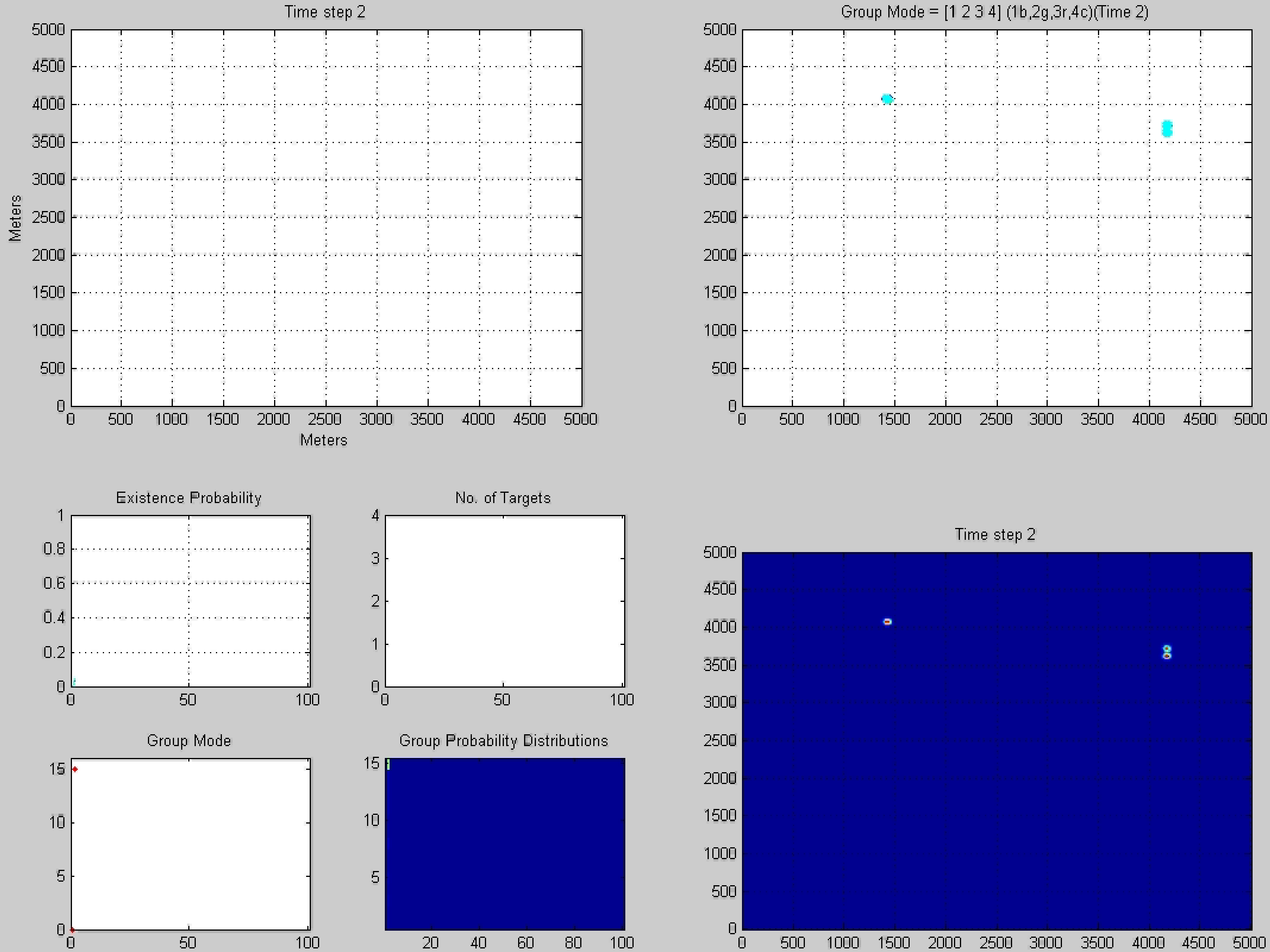
# Simulation Scenario 1

- A single set of tracking result using MCMC-Particles Algorithm.



# Simulation Scenario 1

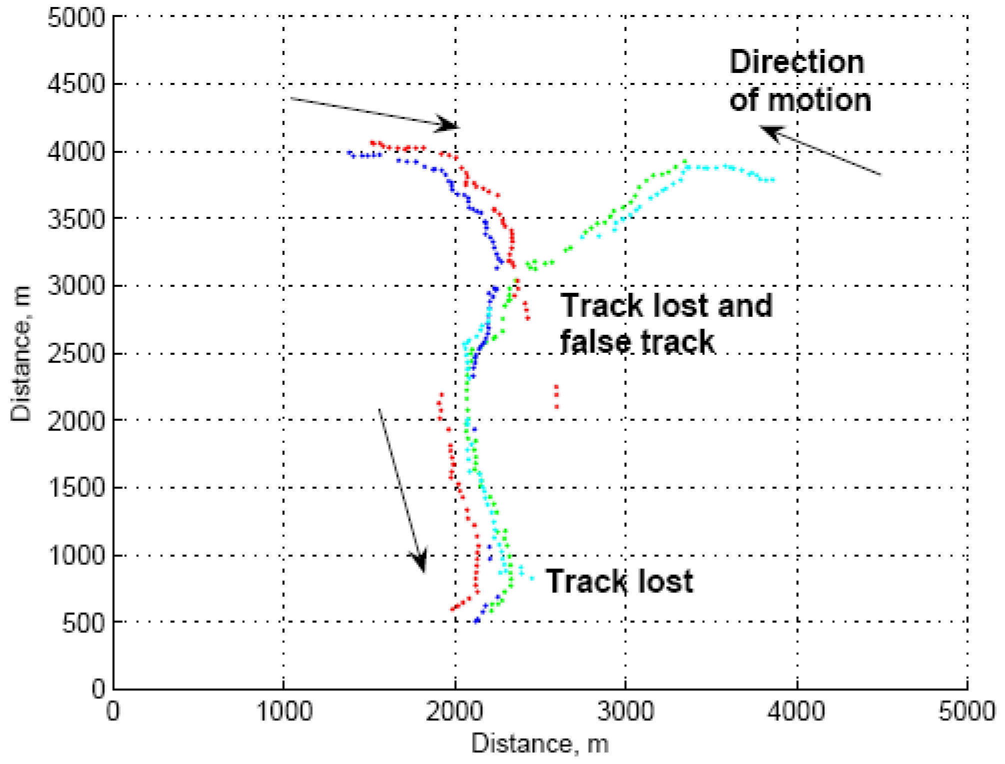




Time step 2
Meters
Meters
Group Mode = [1 2 3 4] (1b,2g,3r,4c)(Time 2)
Existence Probability
No. of Targets
Group Mode
Group Probability Distributions
Time step 2

# Simulation Scenario 1

- Without group tracking, the tracker is more likely to lose track of the targets.



# Simulation Scenario 1

- The average number of targets is significantly more consistency with group tracking as compared with the tracker without group tracking.

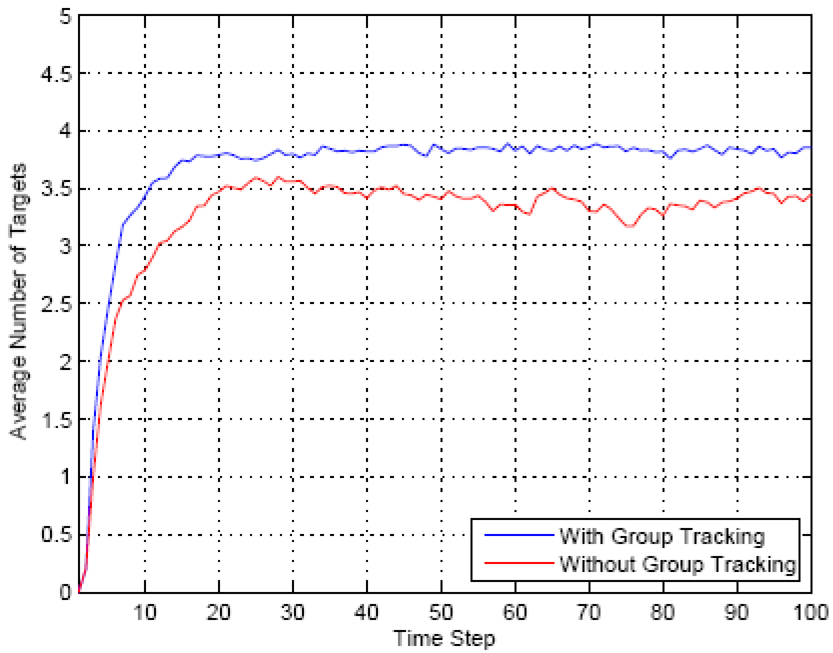
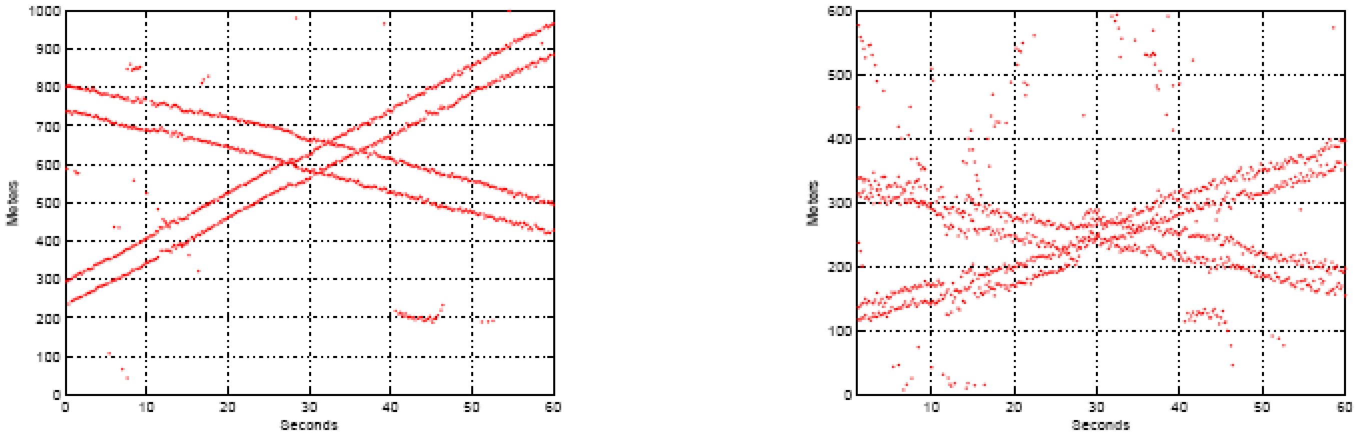



Figure: Average Number of Targets Over 30 Runs

# Introduction

- We adapt the models to track 4 targets in a set of real GMTI radar data
- There are two groups of two targets each (wheeled vehicles) moving along the road in opposite directions. The two groups then move past each other in close proximity.

# GMTI Scenario

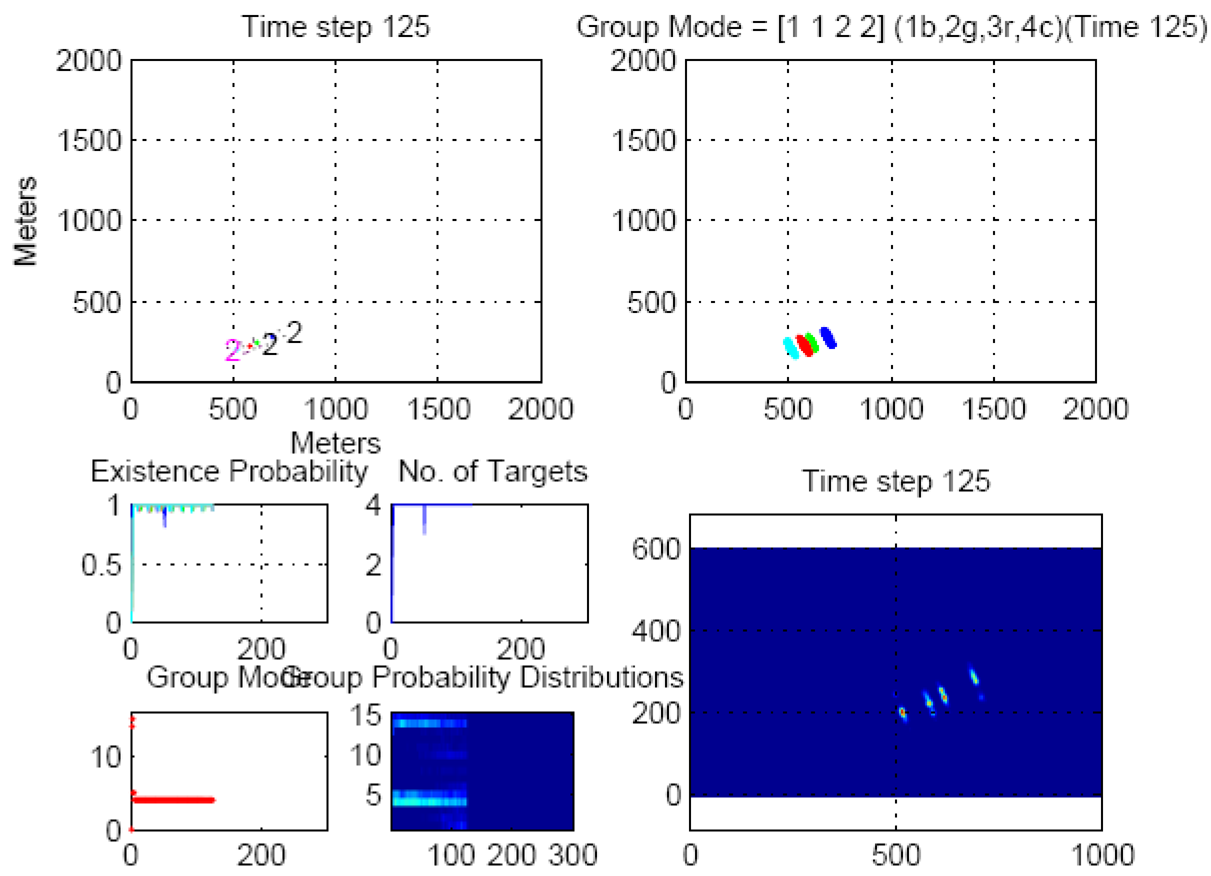


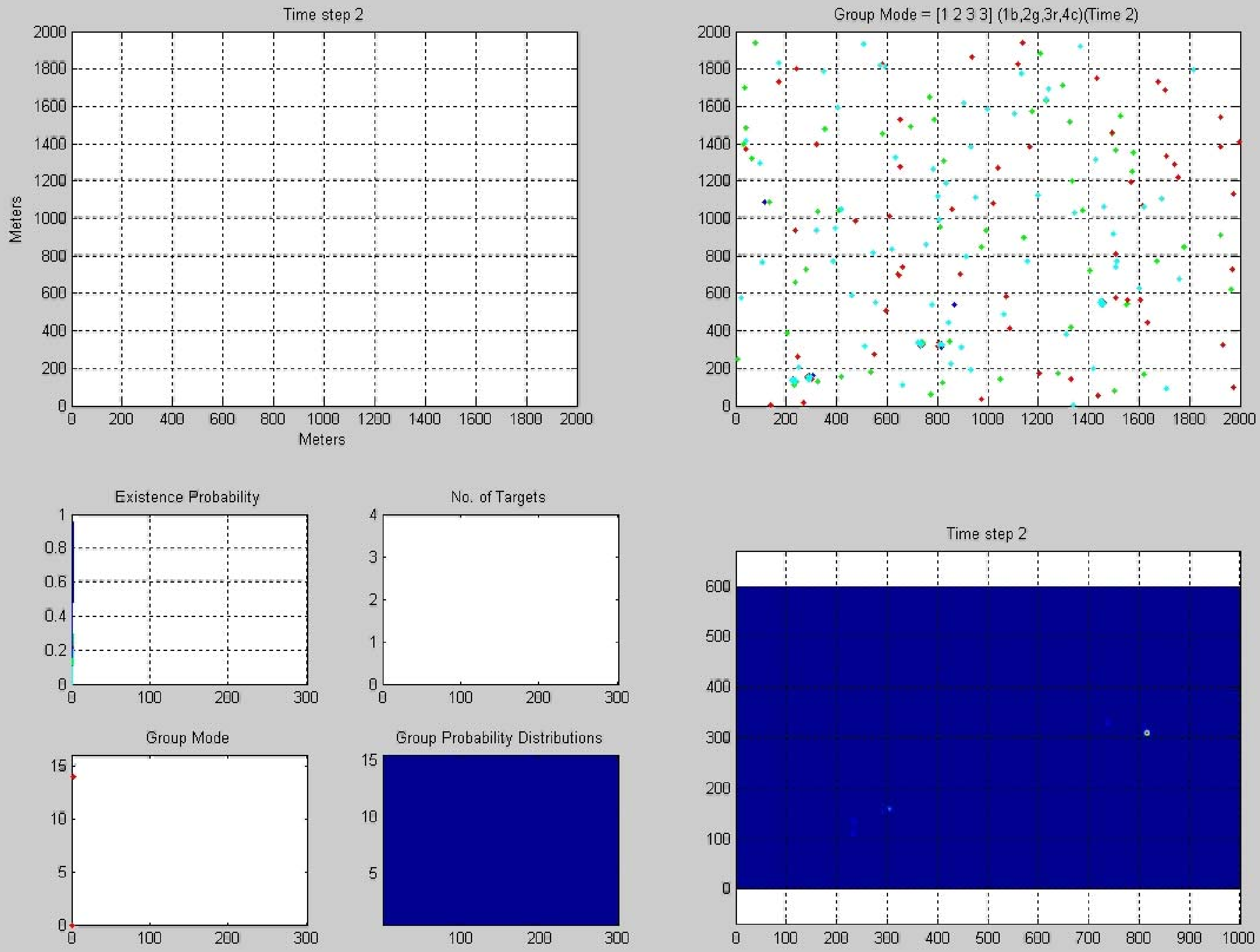

(a) x-coordinate

(b) y-coordinate

Figure: These figures show the x and y coordinates of the actual observations.

# GMTI Scenario





Time step 2
Meters
Group Mode = [1 2 3 3] (1b,2g,3r,4c)(Time 2)
Existence Probability
No. of Targets
Group Mode
Group Probability Distributions
Time step 2

# GMTI Scenario

| Target | Position RMSE with Group Tracking (m) | Position RMSE without Group Tracking (m) | Speed RMSE with Group Tracking (m/s) | Speed RMSE without Group Tracking (m/s) |
|---|---|---|---|---|
| 1 (Blue) | 11.25 | 22.53 | 1.73 | 5.57 |
| 2 (Green) | 17.68 | 20.97 | 2.59 | 3.37 |
| 3 (Red) | 11.20 | 19.86 | 1.23 | 1.87 |
| 4 (Cyan) | NA | NA | NA | NA |

Table: This table compares the RMSE of the tracked position and speed of GMTI Scenario 1 using GPS ground truth. The RMSE is consistently smaller with group tracking than without group tracking.

# Conclusions and Future Directions

- A novel group dynamical model within a continuous time setting and a group structure transition model have been proposed.
- A powerful MCMC-Particle Algorithm has been developed to perform the sequential inference.
- Computer simulations demonstrate the ability of the algorithm to detect and track targets, as well as infer the correct group structure.
- Group tracking using real GMTI radar data also shows improved performances compared to standard multi-target representations
- Current work is extending the methods to larger numbers of targets, also into other domains, including finance.

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[4] Zia Khan, Tucker Balch, and Frank Dellaert, *MCMC-Based Particle Filtering for Tracking a Variable Number of Interacting Targets*, IEEE Transaction on Pattern Analysis and Machine Intelligence **27** (2005), 1805–1819.

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[8] Jaco Vermaak, Simon Maskell, Mark Briers, and Patrick Perez, *A Unifying Framework for Multi-Target Tracking and Existence*, Information Fusion, 2005 8th International Conference on **1** (2005), 25–28.

# Acknowledgements


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